A holomorphic approach to fivebranes

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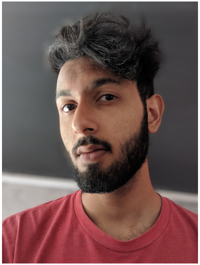
Most work is joint with PhD student Surya Raghavendran at Perimeter Institute.



And some of it with Ingmar Saberi at LMU.

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theory becomes an effective QM system. The state space becomes the $U(1) \times U(1)$-equivariant cohomology of the moduli space of instantons.

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Fivebranes (which are six-dimensional objects before localization) become two-dimensional objects wrapping

$$0 \times C \subset \mathbf{R} \times \mathrm{T}^* C$$

which are chiral defects in the five-dimensional theory.

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- We work with the minimal twist of eleven/six-dimensional supersymmetry. What results is a theory which is ‘less topological’.
- We don’t perform equivariant localization (Ω-background) with respect to the rotation action by $U(1) \times U(1)$.

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- The non-minimal twist, this exists on any manifold of the form $C \times M^4$.

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Degree zero $\overline{\partial}$-cohomology is

$$H^0(\mathbf{C}^3 - 0, \mathcal{O}) = \mathcal{O}(\mathbf{C}^3).$$

The 'singular' part of the OPE lives in $H^2(\mathbf{C}^3 - 0, \mathcal{O})$.

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Theorem (Saberi-W.)

The holomorphic twist of the abelian six-dimensional $\mathcal{N} = (2,0)$ theory has space of fields consisting of:

- *a three-form $\alpha \in \Omega^{2,1}(Z)$ which satisfies the constraint $\partial\alpha = 0$.*
- *a symplectic-valued section $\phi \in \Omega^{0,1}(Z, K^{1/2}) \otimes \mathbf{C}^2$.*

The 'action functional' is

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The moduli space of solutions, modulo gauge symmetries, is a bundle over the intermediate Jacobian variety of Z.

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The idea is that the Hilbert space of the five-dimensional theory on X in the holomorphic twist is the space of de Rham forms on $Bun_G(X)$, which further deforms to de Rham cohomology in the topological twist.

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We now turn to our holographic approach to non-abelian fivebranes.

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Roughly, fields are the cyclic cohomology of category of coherent sheaves on a CY—via HKR we can identify this with polyvector fields and we recover Beltrami equation

$$\overline{\partial}\mu + \frac{1}{2}[\mu, \mu] = 0,$$

as part of the equations of motion.

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- $\mu \in \Pi\Omega^\bullet(\mathbf{R}) \otimes \mathrm{PV}^{1,\bullet}(X)$ required to be divergence-free and locally constant along $\mathbf{R}$.
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$\star$ Caveat: twisted theory carries only an overall $\mathbf{Z}/2$ grading.

If Y is CY4, then the compactification of this theory along a

$$\mathbf{R} \times \mathrm{Tot}(K_Y) \to \mathbf{R} \times \mathbf{R} \times Y = \mathbf{R}^2 \times Y$$

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Theorem (Raghavendran-Saberi-W.)

The algebra of local operators on $\mathbf{R} \times \mathbf{C}^5$ is equivalent to

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We now expand on the program of twisted holography.

Ordinary Koszul duality for associative algebras makes its appearance in QFT when studying (topological) line operators.

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Upshot: $\mathrm{Obs}^!$ is the algebra describing the *universal* line defect.

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Only understand in special examples!

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Back to fivebranes, we look at twisted supergravity on the eleven-manifold

$$M_{bulk} = \mathbf{R} \times \mathrm{Tot}(V \to Z)$$

where: Z is a complex three-fold and $V \to Z$ is a rank two holomorphic vector bundle satisfying $\wedge^2 V \cong K_Z$.

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<u>**Expectation**</u>: the universal theory along the brane = 'theory on stack of $N = \infty$ many (twisted) fivebranes' is

$$(\mathrm{Obs}_{sugra}|_Z)^! = (\pi_* \mathrm{Obs}_{sugra})^!$$

We conjecture that $(\mathrm{Obs}_{sugra}|_Z)^!$ is equivalent to the factorization algebra $\mathrm{Obs}_{\mathfrak{gl}(\infty)}$ on a large number of twisted fivebranes.

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Question: What does twisted holography tell us about

$$\mathrm{Obs}_{\mathfrak{gl}(N)} \quad \text{or} \quad \mathrm{Obs}_{\mathfrak{sl}(N)}?$$

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Every V_j is an irreducible $E(3|6)$ representation. Admits elegant description as quotients of certain $E(3|6)$ Verma modules.

We upgrade this decomposition to a filtration of factorization algebras

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There is a quasi-isomorphism of factorization algebras on the threefold Z (even at the quantum level)

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A good start! What about the next layer?

The next piece has the explicit form

$$\mathcal{F}^{(0)}/\mathcal{F}^{(1)} \simeq \mathrm{C}_\bullet(\mathcal{E}(3|6)_c)$$

where $\mathcal{E}(3|6)$ is a sheaf of super dg Lie algebras on threefold Z which enhances Kac's exceptional super Lie algebra $E(3|6)$.

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Conjecture (Raghavendran-Saberi-W.)

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We also have a putative quantization computed by Witten diagrams in twisted supergravity—appears as turning on a cocycle for the Lie algebra $\mathcal{E}(3|6)$.

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Theorem (Saberi-W.)

$$\left(\widetilde{\mathcal{F}^{(0)}/\mathcal{F}^{(1)}}\ ,\ S \right)$$

is equivalent to the vertex algebra of observables of the chiral sector of the Liouville CFT—that is, the Virasoro algebra of a particular central charge.

The Lie algebra $E(3|6) = \mathcal{E}(3|6)|_{\mathbf{C}^3}$ plays a dual role as an infinite-dimensional enhancement of the twisted superconformal algebra.

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with fugacities (q, t_1, t_2, r).

For $\mathcal{F}^{(0)}/\mathcal{F}^{(1)}$ we find the closed form character of operators

$$\mathrm{PExp}\left[\frac{q^4(t_1^{-1} + t_1 t_2^{-1} + t_2) + q^3(r^2 + r^{-2} + 1) - q^{7/2}(r + r^{-1})(t_1^{-1} + t_1 t_2^{-1} + t_2)}{(1 - t_1^{-1} q)(1 - t_1 t_2^{-1} q)(1 - t_2 q)}\right]$$

which we expect is the refined supersymmetric index of the 6d $\mathcal{N} = (2, 0)$ theory for group $SU(2)$.

One advantage of the holomorphic twist is that the supersymmetric index is easy to compute—it is the character of local operators in the holomorphic twist.

The global symmetry algebra we consider is

$$\mathfrak{gl}(1) \times \mathfrak{sl}(3) \times \mathfrak{sl}(2) \subset \mathfrak{osp}(6|2),$$

with fugacities (q, t_1, t_2, r).

For $\mathcal{F}^{(0)}/\mathcal{F}^{(1)}$ we find the closed form character of operators

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Notice the specialization $q = r^2, t_1 = 1$ is $\mathrm{PExp}\frac{q^2}{1-q}$ = character of Virasoro vacuum.

Given this decomposition of $\widehat{E(5|10)} = \oplus_{j\geq -1} V_j$ into $E(3|6)$-modules we have a systematic way to compute characters in the holomorphic twist at every level of the filtration.

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Specialization $q = r^2, t_1 = 1$ of local character of $\mathcal{F}^{(0)}/\mathcal{F}^{(N-1)}$ agrees with type A_{N-1} $\mathcal{W}$-algebra vacuum character for every N (and in the large N limit as above).

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Future work is to use holography to describe the quantization of the factorization algebras $\mathcal{F}^{(0)}/\mathcal{F}^{(N-1)}$. Also, can we use this program to give a holomorphically twisted construction of class $\mathcal{S}$ theories?