The character of M-theory

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[2103.10271, 2111.07663, and upcoming]
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SETUP

twisted, maximally supersymmetric Yang-Mills theory

- gauge group $U(n)$
- X smooth toric Calabi-Yau three-fold

mathematically, Donaldson-Thomas theory

LESSON #1

using symmetries, compute partition function Z_X
mathematically, K-theoretic equivariant vertex

conjecture: Z_X independent of Coulomb parameters

result: Z_X factorizes
see also [Feyzbakhsh-Thomas]

non-trivial for two reasons:

- higher-rank
- non-empty $H_4(X)$, i.e. $D4$-branes

consequences for geometric engineering

LESSON #2

the space X is non-compact, has isometries G
its G-equivariant volume $\mathcal{F}_X(t, \epsilon)$

- rational function of equivariant parameters ϵ
- power series in Kähler parameters t

satisfies difference equations
encoding action (ψ) of $H^\bullet_{G,\mathrm{cmpt}}(X)$ on $H^\bullet_G(X)$

for example, if $H_4(X) \sim H^2_{\text{cmpt}}(X)$ non-trivial, then

$$\mathcal{F}_X(t,\epsilon) - e^{\epsilon.m}\mathcal{F}_X(t+\psi.m,\epsilon) = \text{analytic in } \epsilon \qquad (1)$$

LESSON # 3

A-twisted gauged linear sigma model
equivariant count quasi-maps $D \to X$

choose stability and boundary conditions
solve equivariant Picard-Fuchs equations

use shift equation to extract enumerative data
after flow to non-linear sigma model