Emergence of Time from Unitary Equivalence

Chen-Te Ma (APCTP)

Pak Hang Chris Lau (Kobe U.)

arXiv: 2204.06366 [hep-th]; Phys.Lett. B795 (2019) 183-187; Int.J.Mod.Phys.A 35 (2020) 18, 2050082.

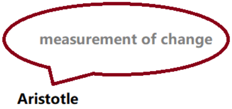
13 July, 2022

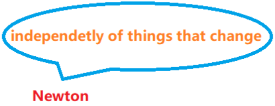
What is Time?

measurement of change

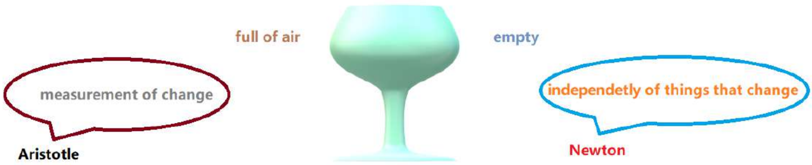
Aristotle

What is Time?

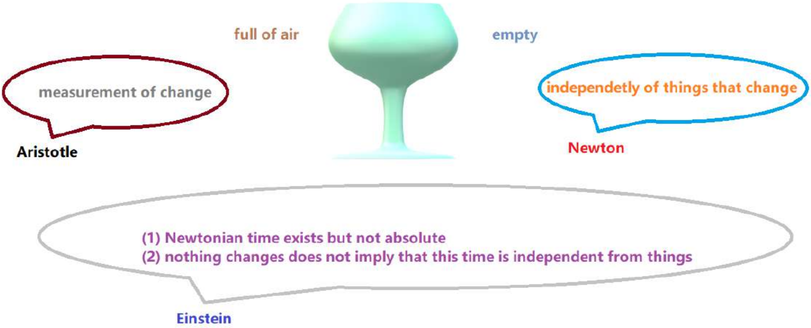

measurement of change
Aristotle


independetly of things that change
Newton

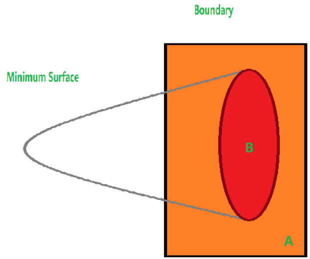
What is Time?

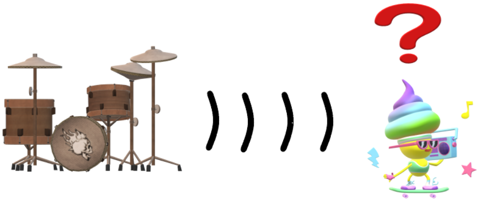

full of air
empty
measurement of change
Aristotle
independetly of things that change
Newton

What is Time?


full of air
empty
measurement of change
Aristotle
independetly of things that change
Newton
(1) Newtonian time exists but not absolute
(2) nothing changes does not imply that this time is independent from things
Einstein

Holographic EE





Unitary Equivalence

- entanglement spectrum should provide the distinguishable constraint

Unitary Equivalence

- entanglement spectrum should provide the distinguishable constraint
- Eigenvectors from Eigenvalues for an Hermitian matrix [P. B. Denton, S. J. Parke, T. Tao, X. Zhang 2019]

Unitary Equivalence

- entanglement spectrum should provide the distinguishable constraint
- Eigenvectors from Eigenvalues for an Hermitian matrix [P. B. Denton, S. J. Parke, T. Tao, X. Zhang 2019]
- the unitary equivalence $\hat{H}_{\mathrm{mod}} = \beta \hat{U} \hat{H}_0 \hat{U}^\dagger$ inspired by the TFD state [D. L. Jafferis, L. Lamprou 2020]

$$|\Psi\rangle \sim \sum_m e^{-\frac{\beta}{2} E_m} \hat{U} |m\rangle \otimes |m\rangle, \tag{1}$$

where E_m is the energy spectrum, $\hat{U}$ is a unitary operator, and $\hat{H}_{\mathrm{mod}} \equiv -\ln \mathrm{Tr}_2 |\Psi\rangle\langle\Psi|$

- the $\hat{H}_0$ is the subsystem's Hamiltonian of the total Hamiltonian

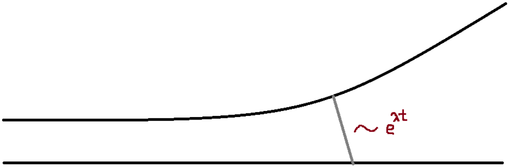
$$\hat{H} = \hat{H}_0 \otimes \hat{I} - \hat{I} \otimes \hat{H}_0 \tag{2}$$

Chaos Bound

- the exponent of the OTOC

$$\langle\Psi|e^{-\frac{\beta}{4}\hat{H}}\hat{U}_1^{\dagger}(t)e^{-\frac{\beta}{4}\hat{H}}\hat{U}_2^{\dagger}(0)e^{-\frac{\beta}{4}\hat{H}}\hat{U}_1(t)e^{-\frac{\beta}{4}\hat{H}}\hat{U}_2(0)|\Psi\rangle\sim e^{\lambda t},\quad t\to\infty, \tag{3}$$

- Conjecture: The saturation of the chaos bound ($\lambda=2\pi/\beta$) in a unitary boundary theory implies AdS Einstein gravity [J. Maldacena, S. H. Shenker, D. Stanford 2016].



Modular Chaos Bound

- The exponent of the Loschmidt echo with a modular Hamiltonian (LEMH)

$$|\langle\psi_0|e^{i\hat{H}_{\mathrm{mod},2}t}e^{-i\hat{H}_{\mathrm{mod},1}t}|\psi_0\rangle|^2 \sim e^{\lambda t},\ t\to\infty,$$
$$\hat{H}_{\mathrm{mod},2} \equiv \hat{H}_{\mathrm{mod},1} + \delta\hat{H}_{\mathrm{mod}}, \tag{4}$$

provides the modular chaos bound ($\lambda = 2\pi$) [J. de Boer, L. Lamprou 2020].

- The saturation implies the most sensitive infinitesimal perturbation on a quantum state.

OTOC and LE: Fermionic Theory

- fermionic coherent state $|\psi\rangle$
- consider the operators

$$\hat{U}_{(\alpha,\gamma)}(0) \equiv e^{i(\alpha^* \hat{c} + \hat{c}^\dagger \alpha) + (\gamma^* \hat{t} - \hat{t}^\dagger \gamma)} e^{-\frac{1}{2}\alpha^* \alpha}, \tag{5}$$

where

$$\hat{t} \equiv \frac{\partial}{\partial \psi^*}; \qquad \hat{t}^\dagger \equiv \frac{\partial}{\partial \psi} \tag{6}$$

- separated into two parts $\hat{U}(0) = \hat{S}(0)\hat{T}(0)$, where

$$\hat{S}(0) \equiv e^{i\alpha^* \hat{c}} e^{i\hat{c}^\dagger \alpha}, \;\; \hat{T}(0) \equiv e^{\gamma^* \hat{t} - \hat{t}^\dagger \gamma} \tag{7}$$

- the $\hat{T}(0)$ operator acts on the coherent state as a shift to the parameter
- inspired from the group element of the Heisenberg group

$$\hat{U}(q_1, q_2) \equiv \exp(iq_1 \hat{X}(0) + iq_2 \hat{P}(0)) \tag{8}$$

OTOC and LE: Duality

- total system Hamiltonian

$$\hat{H} = \hat{H}_1 \otimes \hat{I}_2 - \hat{I}_1 \otimes \hat{H}_2 + \sum_{\alpha} \hat{H}'_{\alpha} \otimes \hat{H}''_{\alpha}. \tag{9}$$

-

$$\begin{aligned} G_{AV,4} &= \int_{\psi} \int_{\hat{U}_1, \hat{U}_2} \langle\psi| \hat{U}_1^{\dagger}(t_1) \hat{U}_2^{\dagger}(0) \hat{U}_1(t_1) \hat{U}_2(0) |\psi\rangle \\ &= \frac{1}{N_1^2} \sum_{\hat{P}_1, \hat{P}'_1} \left| \int_{\psi_2} \langle\psi_2| \, e^{i(\hat{H}_2+\hat{P}_1)t_1} e^{-i(\hat{H}_2+\hat{P}'_1)t_1} \, |\psi_2\rangle \right|^2, \end{aligned} \tag{10}$$

where we insert the operator $\hat{U}_1(t_1)$ localized on the 1-th site at time t_1, and the N_1 is the number of operators $\hat{P}_1$

Modular Chaos Bound=Chaos Bound

- regularized thermal expectation value by inserting thermal factors (or shifting the time parameters) as

$$t_j \to t_j - i\frac{\beta}{4}(2-j)\ \forall j = 1, \cdots, 4 \tag{11}$$

- introduce a modular parameter $s \equiv t/\beta$ and replace the subsystem Hamiltonian with the modular Hamiltonian in the formula of LE using unitary equivalence

Modular Chaos Bound=Chaos Bound

- regularized thermal expectation value by inserting thermal factors (or shifting the time parameters) as

$$t_j \to t_j - i\frac{\beta}{4}(2-j)\ \forall j = 1, \cdots, 4 \tag{11}$$

- introduce a modular parameter $s \equiv t/\beta$ and replace the subsystem Hamiltonian with the modular Hamiltonian in the formula of LE using unitary equivalence
- (1) the LE expressed by the modular Hamiltonian probes modular chaos; (2) the dual correlator with the total Hamiltonian encodes information about chaos

Modular Chaos Bound=Chaos Bound

- regularized thermal expectation value by inserting thermal factors (or shifting the time parameters) as

$$t_j \to t_j - i\frac{\beta}{4}(2-j)\ \forall j = 1, \cdots, 4 \tag{11}$$

- introduce a modular parameter $s \equiv t/\beta$ and replace the subsystem Hamiltonian with the modular Hamiltonian in the formula of LE using unitary equivalence
- (1) the LE expressed by the modular Hamiltonian probes modular chaos; (2) the dual correlator with the total Hamiltonian encodes information about chaos
- the dual implies "Modular Chaos Bound=Chaos Bound" with the constraint of unitary equivalence

Thank you!