Revisiting Heterotic ALE Instantons: F-theory, 2-groups, and T-duality

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With Michele Del Zotto, Paul-Konstantin Oehlmann – 2207-xxxx

Outline

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6D Little String Theory (LST)

Has the following features - between SUGRA and SCFT

- **Decoupled** from **gravity**
- String excitations have an intrinsic **string tension**
  $\rightarrow$ key in the dynamics of the theory
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- Compactified on **non compact** Clabi-Yau threefold CY_3
- **Negative semi definite** dirac pairing, $\eta^{IJ} \geq 0$
  $\rightarrow$ has a **zero curve** $\Sigma_0 = \sum N_I \Sigma_I$
- All known LSTs can be realised within **F-theory**

  [Bhardwaj, Del Zotto, Heckman, Morrison, Rudelius, Vafa '15]

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$\Rightarrow$ Construct Heterotic ALE instantons and identify T-duals.
Details of geometric engineering : See Oehlmann's talk

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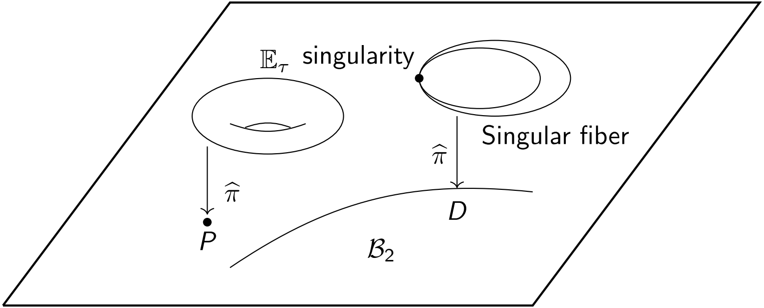
- It is described by a singular elliptic Calabi-Yau fibration $\pi\colon Y_3 \twoheadrightarrow \mathcal{B}_2$, which admits a smooth, flat and crepant resolution $\widehat{Y}_3$:



Geometrizing heterotic ALE instantons

Approached by F/Heterotic duality

- In 8D, $E_8 \times E_8$ **Heterotic string** on $\mathbb{T}^H_2$ is **dual to F-theory** on an elliptic K3 with a stable degeneration limit into two dP_9s.

$$\begin{array}{ccc} \mathbb{T}^2_f & \longrightarrow & K3 \\ & & \big\downarrow f \\ & & \mathbb{P}^1 \end{array} \quad \rightarrow \quad \begin{array}{ccc} \mathbb{T}^2_f & \longrightarrow & dP_9 \vee_{\mathbb{T}^2_H} dP_9 \\ & & \big\downarrow f \\ & & \mathbb{P}^1 \end{array}$$

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- N **Heterotic instantons** are realized as a $\mathbb{C}^2/\mathbb{Z}_N$ singularity where the two -1 curves of P_1 bases intersect.

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- The six dimensional reduction of the Heterotic string on $\mathbb{T}_2^H \to \mathbb{C}$ is equivalent to F-theory on $K3 \to \mathbb{C}$
- In practise, we construct Y_3 by toric method. Huang Taylor'18, Anderson, Gao, Gray, Lee'16

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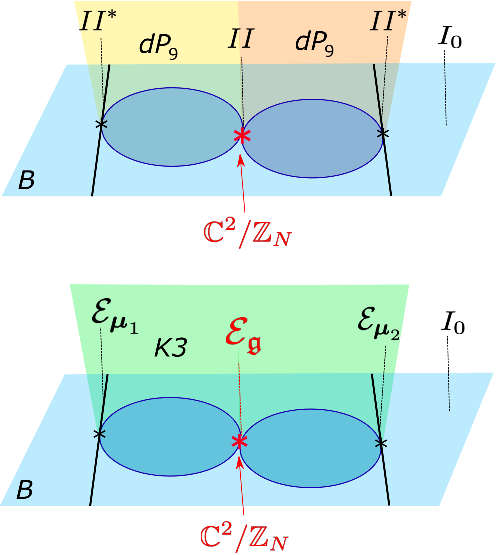
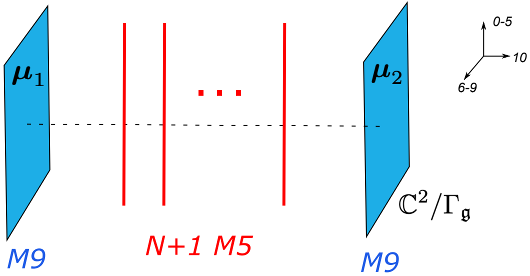


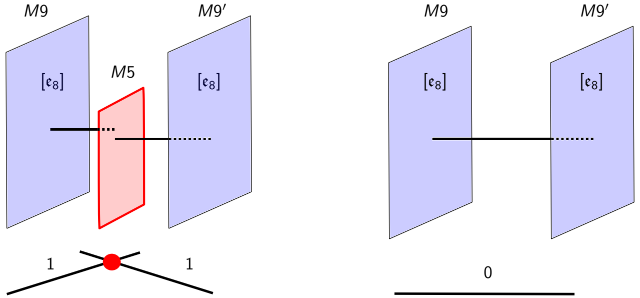
Hořava-Witten duality

- Heterotic instantons $\leftrightarrow$ N M5 branes parallel to the two M9s
- two copies of E8 $\leftrightarrow$ two M9s



Heterotic strings probing ADE singularities

- Up: M9 brane and their connecting M2 branes/strings
- below: local F-theory curve configuration



Outline

Basic ingredients to cook a LST

- Description of generalized quiver theories:

$$\mathcal{T}(\boldsymbol{\mu_1}, \mathfrak{g}) \overset{\mathfrak{g}}{\text{———}} \mathcal{T}_N(\mathfrak{g}, \mathfrak{g}) \overset{\mathfrak{g}}{\text{———}} \mathcal{T}(\boldsymbol{\mu_2}, \mathfrak{g})$$

$$\underbrace{1\,2\,2\,2\,\cdots\,2\,2\,1}_{N+1}$$

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- $\mathcal{T}(\boldsymbol{\mu_a}, \mathfrak{g})$: M9-M5 system with $\mathbb{C}^2/\Gamma_{\mathfrak{g}}$
  $\leftrightarrow$ determined by choice $\boldsymbol{\mu_a} : \Gamma_{\mathfrak{g}} \to E_8$
  $\to$ Zero form flavor symmetry.
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- $\mathcal{T}_N(\mathfrak{g}, \mathfrak{g})$: N M5 branes probing a $\mathbb{C}^2/\Gamma_{\mathfrak{g}}$ singularity;
  $\leftrightarrow$ determined by $[G] - [G]$ conformal matter.

Tensor branch from conformal matter Zotto, Heckman, Tomasiello, Vafa '14

- M9-M5 Distance : parametrized by vev of (1,0) tensormultiplets $\leftrightarrow$ fractions of the intersection point on Hoařva-Witten wall

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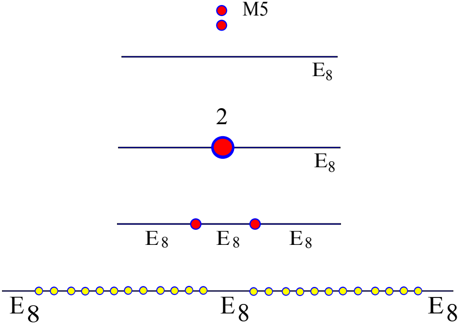
- M5-M5 Distance : parametrized by vev of (1,0) tensormultiplets $\leftrightarrow$ fractions of M5-branes Zotto, Heckman, Tomasiello, Vafa '14



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2-group symmetry and T-duals

- LSTs have a continuous **two-group symmetry** [Córdova,Dumitrescu, Intriligator '18]

  $\leftrightarrow$ Different form-degree symmetries mix to higher groups:

$$\left(P^{(0)} \times SU(2)_R^{(0)} \times \prod_a F_a^{(0)}\right) \times_{\widehat{\kappa}_P, \widehat{\kappa}_R, \widehat{\kappa}_{F_a}} U(1)_{LST}^{(1)}.$$

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- $\widehat{\kappa}_P$, $\widehat{\kappa}_R$ and $\widehat{\kappa}_{F_a}$ measure the mixing of the symmetries
  $\leftrightarrow$ and Dim(Cb) be **matched for T-duals** of LSTs.

$$\widehat{\kappa}_F = -\sum_{I=1}^{r+1} N_I \eta^{IA} \qquad \widehat{\kappa}_R = \sum_{I=1}^{r+1} N_I h^\vee_{\mathfrak{g}_I} \qquad \widehat{\kappa}_P = -\sum_{I=1}^{r+1} N_I (\eta^{II} - 2)\,.$$

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- 5d M-theory on CY_3 has multiple F-theory uplifts at 6d
  $\leftrightarrow$ encoded by different ellipic fibrations in CY_3.

Found : Mutiple T-dual LSTs

A concrete CY_3 with **three elliptic fibrations** $\leftrightarrow$ **T-dual triples**:

1. An $\mathfrak{su}_2^{M+1}$ gauge group given as the chain

$$[\mathfrak{so}_{16}^{-}] \; \underbrace{\overset{\mathfrak{su}_2}{1} \;\; \overset{\mathfrak{su}_2}{2} \; \ldots \; \overset{\mathfrak{su}_2}{2} \;\; \overset{\mathfrak{su}_2}{1}}_{\times M+1} \; [\mathfrak{so}_{16}^{+}] \,. \tag{1}$$

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3. A single tensor theory with

$$[\mathfrak{su}_{16}] \;\; \overset{\mathfrak{su}_{2M+1}}{0} \;\; , \qquad (3)$$

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More results

$\mathfrak{g}$	$F(\mu_1,\mu_2)$	Theory description
E_6^M	$[E_6\times E_6\times SU(3)]/\mathbb{Z}_3$	$[SO(7)]\ \overset{\mathfrak{sp}_{M-1}}{1}\ \overset{\mathfrak{so}_{4M+4}}{4}\ \overset{\mathfrak{sp}_{3M-3}}{1}\ \overset{\mathfrak{su}_{4M}}{2}\ \overset{\mathfrak{su}_{2M+6}}{2}\ [SU(12)]$
	$E_8\times E_7$	$[SO(28)]\ \overset{\mathfrak{sp}_{M+7}}{1}\ \overset{\mathfrak{so}_{4M+16}}{4}\ \overset{\mathfrak{sp}_{3M+1}}{1}\ \overset{\mathfrak{su}_{4M+2}}{2}\ \overset{\mathfrak{su}_{2M+2}}{2}\ [SU(2)]$
E_7^M	$[E_7\times E_7\times SU(2)]/\mathbb{Z}_2$	$[SU(16)]\ \overset{\mathfrak{su}_{2M+10}}{2}\ \overset{\mathfrak{su}_{4M+4}}{2}\ \overset{\mathfrak{su}_{6M-2}}{2}\ \overset{\mathfrak{sp}_{4M-4}}{1}\ \overset{\mathfrak{so}_{4M+4}}{4}$
E_8^M	$E_7\times E_7$	$[SO(24)]\ \overset{\mathfrak{sp}_{M+7}}{1}\ \underset{[N_F=2]}{\overset{\mathfrak{so}_{4M+20}}{4}}\ \overset{\mathfrak{sp}_{3M+3}}{1}\ \overset{\mathfrak{so}_{8M+8}}{4}\ \overset{\mathfrak{sp}_{5M-3}}{1}\ \overset{\overset{\mathfrak{sp}_{3M-5}}{1^*}}{\overset{\mathfrak{so}_{12M-4}}{4^*}}\ \overset{\mathfrak{sp}_{4M-4}}{1}\ \overset{\mathfrak{so}_{4M+4}}{4}$
$SO(4N+5)$	$E_8\times E_8$	$[E_8]\ \underbrace{\cdots}_{\text{symmetric}}\ \underset{[N_F=1]}{\overset{\mathfrak{so}_{4N+5}}{4}}\ \underbrace{\overset{\mathfrak{sp}_{2N-2}}{1}\ \overset{\mathfrak{so}_{4N+3}}{4}\ \ldots\ \overset{\mathfrak{sp}_{2N-2-k}}{1}\ \overset{\mathfrak{so}_{4N+3-2k}}{4}\ \ldots\ \overset{\mathfrak{sp}_1}{1}\ \overset{\mathfrak{so}_9}{4}}_{k=0,\cdots,2N-3}\ 1\ \overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{su}_2}{2}\ 2\ 1\ [E_8]$
$SO(4N+7)$	$E_8\times E_8$	$[E_8]\ \underbrace{\cdots}_{\text{symmetric}}\ \underset{[N_F=1]}{\overset{\mathfrak{so}_{4N+7}}{4}}\ \underbrace{\overset{\mathfrak{sp}_{2N-1}}{1}\ \overset{\mathfrak{so}_{4N+5}}{4}\ \ldots\ \overset{\mathfrak{sp}_{2N-1-k}}{1}\ \overset{\mathfrak{so}_{4N+5-2k}}{4}\ \ldots\ \overset{\mathfrak{sp}_1}{1}\ \overset{\mathfrak{so}_9}{4}}_{k=0,\cdots,2N-2}\ 1\ \overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{su}_2}{2}\ 2\ 1\ [E_8]$

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Summary and Conclusion

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- Construct various **LST** : $F_1 \times F_2$ probing $\mathfrak{g}$ models on non compact Calabi-Yau threefold
- **Verify** new T-duals via **2-group data matching**

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Outlook

- Classification of heterotic strings probing ADE singularity
- Twisted compactification, genus one fibration
- More T-dual system via possible elliptic fibrations of a fixed K3