Holomorphic boundary conditions for topological field theories via branes in twisted supergravity

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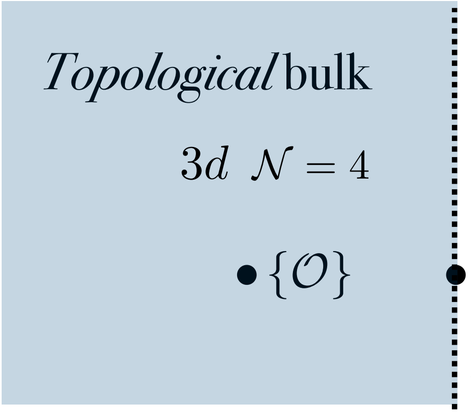
University of Warsaw

Motivation and Objective:

Twisting $3d\ \mathcal{N}=4$ theories **with** boundaries:



Holomorphic boundary

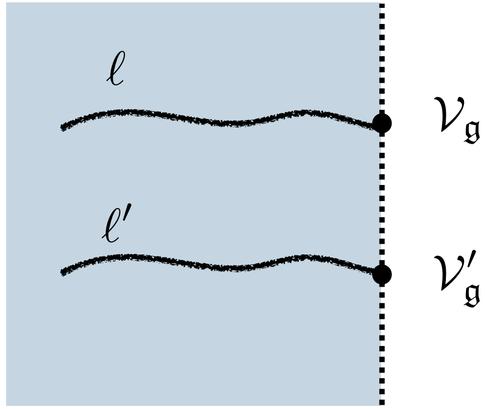
$2d\ \mathcal{N}=(0,4)$

$\mathfrak{g}_{\{\mathcal{O}_\partial\}}$:

vertex operator algebra (VOA) of bdy operators

- Significance:

  Study bulk TFT via boundary VOA data



Costello - Creutzig -Gaiotto '18

- Central issue:

  Compatibility of boundary with bulk topological twist via appropriate *deformation*

Gaiotto-Costello '18

Gaiotto-Rapčak '18

In this talk:

Show how topological twist is implemented as a deformation of a holomorphically twisted theory, by engineering the theories in brane world volumes in a particular background

Motivation and Objective:

Topological Twisting:

- Choice of twisting homomorphism:

$$\mathfrak{h}: \ \mathrm{Spin}(n) \to \mathrm{Spin}'(n) \subset (\mathrm{Spin}(n) \times G_R)$$

$$\mathcal{Q} : \mathrm{Spin}'(n) - \mathrm{inv.}, \quad \mathcal{Q}^2 = 0$$

- Pass on $\mathcal{Q}$ − cohomology:

  Restrict to $\mathcal{Q}$ −closed operators

  $\mathcal{Q}$ − exact quantities vanish

- Focus on a protected (BPS) subsector of the theory
- P_μ become $\mathcal{Q}$ − exact: *Topological* theory

$$\langle \mathcal{O}_1 \mathcal{O}_2 .. \rangle \neq f(\{x\})$$

- Further twists?

  Holomorphic twist: $\partial/\partial\bar{z}$ become $\mathcal{Q}$ − exact

$$\langle \mathcal{O}_1 \mathcal{O}_2 .. \rangle = g(z)$$

$$\textit{Holomorphic} \xrightarrow{\ ?\ } \textit{Topological}$$

Outline:

- 3d N=4 theories with boundaries:

  Topological and Holomorphic twists

  Boundaries, deformations and compatibility

- Brane engineering:

  Bulk and boundary theory

  Twisting and background geometry

- Twisted IIB supergravity:

  Holomorphic brane engineering

  Background deformations and compatibility

- Summary, future directions and ongoing work

$3d\ \mathcal{N}=4$ with boundaries:

Superconformal algebra:

$$\mathfrak{g}(3,\mathcal{N}=4)=\mathfrak{osp}(4|4)\supset\mathfrak{so}(3,2)\times\mathfrak{so}(4)_R$$

$$Q\in[\mathbf{2}]^{(\mathbf{2},\mathbf{2})} \qquad SO(4)_R\simeq SU(2)_C\times SU(2)_H$$

$$\{Q_\alpha^{A\dot{A}},Q_\beta^{B\dot{B}}\}=\epsilon^{AB}\epsilon^{\dot{A}\dot{B}}P_{\alpha\beta}$$

- 3d N=4 Hypermultiplet

Scalars: $(q,\tilde{q})\in[\mathbf{1}]^{(\mathbf{1},\mathbf{2})}$

Fermions: $(\psi,\tilde{\psi})\in[\mathbf{2}]^{(\mathbf{2},\mathbf{1})}$

Moduli space:

$$\mathcal{M}=\bigcup_n\mathcal{C}_n\times\mathcal{H}_n$$

- $\mathcal{M}_C\simeq\mathcal{C}\times\{0\}$
- $\mathcal{M}_H\simeq\{0\}\times\mathcal{H}$

Twisting:

- $\mathfrak{h}_{C,H}:SU(2)_L\to SU(2)_L\times SU(2)_{C,H}$
- $\mathcal{Q}:\mathcal{Q}^2=0,$

- Three types of nilpotent supercharges:

$$\{\mathcal{Q}_C,\mathcal{Q}_H,\mathcal{Q}_{\text{hol}}\}$$

$3d\ \mathcal{N} = 4$ with boundaries:

Consider topological supercharges as *deformations* of the holomorphic one:

$$\mathcal{Q}_H = \mathcal{Q}_{\text{hol.}} + \zeta Q_H \qquad\qquad \mathcal{Q}_C = \mathcal{Q}_{\text{hol.}} + \zeta Q_C$$

Gaiotto-Costello '18

$\longrightarrow$ Apply to construction of appropriate boundary conditions!

- $\mathcal{N} = (0,4)$: Compatible with bulk *holomorphic* but *not topological* twist

  Deformable to be *compatible* with bulk topological twist

- $\mathcal{N} = (2,2)$: Compatible with both *holomorphic and topological* twist

Holomorphic twist

$\downarrow$

Deform

$\downarrow$

Topologically twisted theory

$3d\ \mathcal{N}=4$ with boundaries:

- (0,4) b.c for free hypermultiplet:

  Or how deformations of bdy conditions are induced by deformations of the preserved susy:

$$\mathcal{Q}_{\text{hol.}} + \zeta Q_{H,C} \longrightarrow \mathcal{J}_{\perp}^{\text{hol}} + \zeta \mathcal{J}_{\perp}^{H,C} = 0$$

Gaiotto-Rapčak '18

- C : $\tilde{\mathcal{B}}_D \phi$: (0,4) Dirichlet b.c can be deformed to be compatible with C-twist
- H : $\tilde{\mathcal{B}}_N \phi$: (0,4) Neumann b.c can be deformed to be compatible with H-twist

- $S_\partial \mapsto S_\partial + \mathcal{S}$, $\delta_{Q_{\text{hol}}} \mathcal{S} = \mathcal{J}_\partial^{Q_{\text{hol.}}}$

Defo preserving H,C-deformed susy if: $\delta_{Q_{\text{hol}}} \tilde{\mathbb{S}} = \mathcal{J}_\partial^{Q_{H,C}}$

N: H-twist $\tilde{\mathbb{S}} = \mathbb{S}_{Sb}$

D: C-twist $\tilde{\mathbb{S}} = \mathbb{S}_{Fc}$

Symplectic Boson VOA

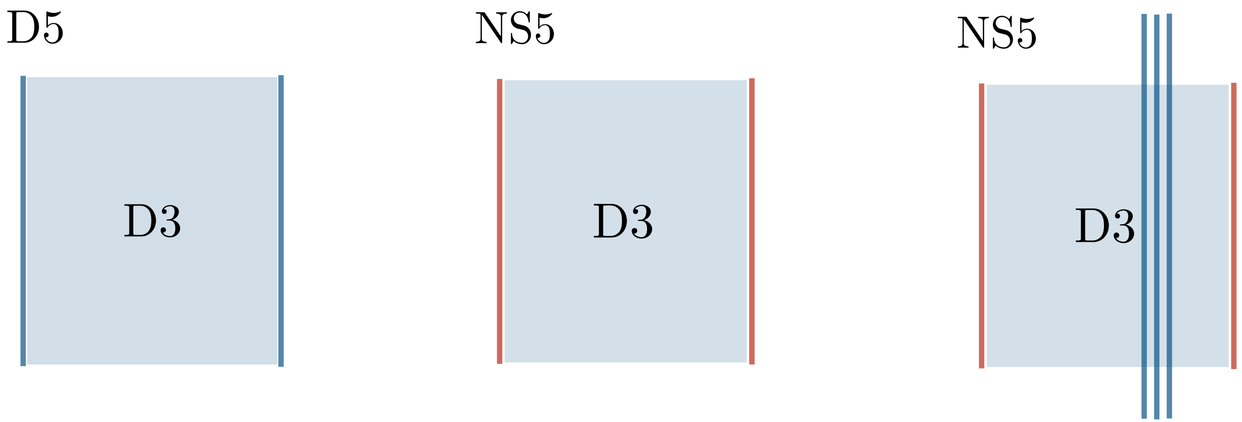
$$X(z)Y(w) \sim \frac{1}{z-w}$$

Symplectic Fermion VOA

$$x(z)y(w) \sim \frac{1}{(z-w)^2}$$

Brane engineering:

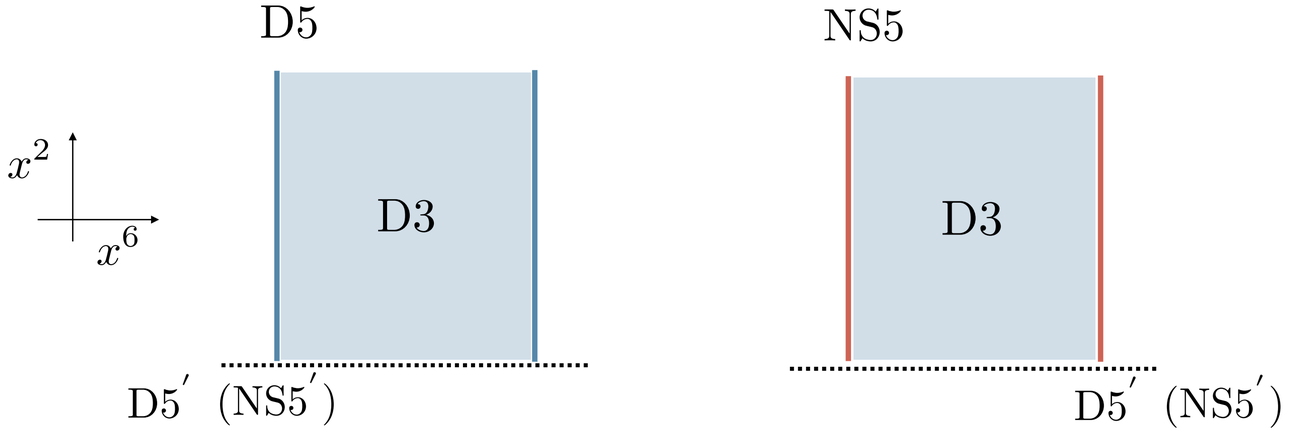
- 3d N=4 theories can be realized as brane world volume theories:



	0	1	2	3	4	5	6	7	8	9
D3	•	•	•				•			
D5	•	•	•					•	•	•
NS5	•	•	•	•	•	•				

Hanany-Witten '96

- Boundaries realized from extra IIB five-branes:



$\mathcal{N} = (0, 4)$

D5′	•	•		•	•	•	•			
NS5′	•	•					•	•	•	•

Hanany-Okazaki '18

Brane engineering:

Twisting homomorphisms:

- Appropriate choice of target space geometry:

  Bershadsky, Vafa, Sadov '96

$$M_3 \hookrightarrow T^*_{H,C} M_3 \subset \mathrm{CY}_3$$

	0	1	2	3	4	5	6	7	8	9
D3	•	•	•				•			
D5	•	•	•					•	•	•
NS5	•	•	•	•	•	•				

- Several points remain beyond reach:

  Q-cohomology?

  Engineering of holomorphic twist?

  Deformations of partial twists?

$$\Gamma_{\mu\nu} \to \Gamma_{\mu\nu} + \Gamma_{(\mu+3)(\nu+3)},$$

$$\mu, \nu = 0, 1, 2$$

$$\Gamma_{\mu\nu} \to \Gamma_{\mu\nu} + \Gamma_{(\mu+7)(\nu+7)},$$

Twisted IIB supergravity:

Definition:

Supergravity background where, the bosonic ghost of local supersymmetry acquires non-vanishing v.e.v

Costello, Li '16

- Gauging local supersymmetry $\longrightarrow$ bosonic ghost $q \in C^\infty(\mathbb{R}^{10}, S)$

- Supergravity EoM satisfied for:

  q is cov. const. spinor : $q = \mathcal{Q}$

  $\Gamma(q,q) = 0$ with $\Gamma : S \otimes S \to \mathbb{R}^{10}$

- Twisted SQFT

  SQFT coupled to twisted sugra background : $d_{\mathrm{BRST}} \to d_{\mathrm{BRST}} + \mathcal{Q}$

Different choice of bosonic ghosts $\longrightarrow$ different twists

Twisted IIB supergravity:

Conjecture:

$$\boxed{\text{Holomorphic twist of IIB supergravity : BCOV theory}}$$

Costello, Li '16

- BCOV theory (Kodaira-Spencer): closed string field theory of the topological B-model

$\longrightarrow$ Complex structure deformations

Bershadsky, Cecotti, Ooguri, Vafa '94

- $X = \mathrm{CY}_d$: $\quad \mathrm{PV}^{i,j}(X) = \Omega^{0,j}(X, \wedge^i \mathrm{T}X) \qquad \mathrm{PV}^{i,j}(X) \xrightarrow{\Omega_X} \Omega^{d-i,j}(X)$

- Space of fields: $\mathcal{E}_{\mathrm{BCOV}} = \left(\mathrm{PV}^{\bullet,\bullet}(X)[t][2], \bar{\partial} + t\partial\right)$

- Subspace of <u>propagating</u> fields: "Minimal BCOV"

$$\mathcal{E}_{\mathrm{BCOV}} \supset \mathcal{E}_{\mathrm{mBCOV}} = \left(\bigoplus_{i+j<d} t^i \mathrm{PV}^{j,\bullet}(X)[2], \bar{\partial} + t\partial \right)$$

Twisted IIB supergravity:

- Focus on flat space:

Holomorphically twisted IIB supergravity on $\mathbb{R}^{10}$ - mBCOV theory on $\mathbb{C}^5$

Supersymmetry and BCOV fields:

<u>10d $\mathcal{N}=(2,0)$</u> $\quad \mathfrak{g}_{IIB} : \mathfrak{so}(10,\mathbb{C}) \oplus V \oplus \Pi(S_+ \otimes \mathbb{C}^2) \oplus \{...\}$ and $\psi \in S_+ : \Gamma(\psi,\psi) = 0$

<u>Twisted background:</u> $\quad \mathcal{Q}_\psi \in \mathfrak{g}_{IIB}, \quad \mathcal{Q}^2 = 0 \longrightarrow \boxed{q = \mathcal{Q}_\psi}$

<u>Residual supersymmetry</u>: $\mathcal{Q}_\psi$-cohomology of the IIB algebra: $\quad \mathfrak{g}_{IIB}^{\mathcal{Q}_\psi} : \mathfrak{sl}(5,\mathbb{C}) \oplus \mathbb{C}^5 \oplus \Pi(S_+ \otimes v) \oplus \{...\}$

Susy present in mBCOV:

$$\mathfrak{R} : \mathcal{H}^\bullet(\mathfrak{g}_{IIB}, \mathcal{Q}_\psi) \to \mathcal{E}_{mBCOV}$$

$$dz_i \to z_i \in \mathrm{PV}^{0,0}$$

$$\partial_{z_i} \wedge \partial_{z_j} \to \partial_{z_i} \wedge \partial_{z_j} \in \mathrm{PV}^{2,0}(\mathbb{C}^5)$$

- Odd elements of $\mathfrak{g}_{IIB}^{\mathcal{Q}_\psi}$ transform as:
  <u>1-forms</u> and <u>bivectors</u> under $\mathfrak{sl}(5,\mathbb{C})$

Costello, Li '16

Twisted IIB supergravity:

Holomorphic Hanany-Witten: the case of the 3d hypermultiplet

bulk:	$\mathbb{C}_{z_1}$	$\times$	$\mathbb{C}_{w_2}$	$\times$	$\mathbb{C}_{z_2}$	$\times$	$\mathbb{C}_{w_1}$	$\times$	$\mathbb{C}_{z_3}$
	$\cong$		$\cong$		$\cong$		$\cong$		$\cong$
	$\mathbb{R}^2_{01}$	$\times$	$\mathbb{R}^2_{26}$	$\times$	$\mathbb{R}^2_{34}$	$\times$	$\mathbb{R}^2_{59}$	$\times$	$\mathbb{R}^2_{78}$
D3:	$\mathbb{C}_{z_1}$	$\times$	$\mathbb{C}_{w_2}$	$\times$	0	$\times$	0	$\times$	$0.$

✔ Holomorphic configuration on bulk and brane

- D3 world volume theory: holomorphic Chern-Simons on $\mathbb{C}_{z_1} \times \mathbb{C}_{w_2} \subset \mathbb{C}^5$

$$\mathcal{E}_{\mathrm{D3}} = \Omega^{0,\bullet}(\mathbb{C}^2)[d\bar{z}_2, d\bar{w}_1, d\bar{z}_3]$$

- Next: need to add the D5s in order to get $3d\ \mathcal{N} = 4$ configuration

Twisted IIB supergravity:

- The D5 support is not along a complex submanifold of $\mathbb{C}^5$
- Deform, using the bivector $\partial_{w_1} \wedge \partial_{w_2}$

$$
\begin{array}{lccccccccc}
\text{bulk:} & \mathbb{C}_{z_1} & \times & \mathbb{C}_{w_2} & \times & \mathbb{C}_{z_2} & \times & \mathbb{C}_{w_1} & \times & \mathbb{C}_{z_3} \\
 & \cong & & \cong & & \cong & & \cong & & \cong \\
 & \mathbb{R}^2_{01} & \times & \mathbb{R}^2_{26} & \times & \mathbb{R}^2_{34} & \times & \mathbb{R}^2_{59} & \times & \mathbb{R}^2_{78} \\
\hline
\text{D3:} & \mathbb{C}_{z_1} & \times & \mathbb{C}_{w_2} & \times & 0 & \times & 0 & \times & 0. \\
\text{D5:} & \mathbb{C}_{z_1} & \times & \mathbb{R}_2 \times \partial I_6 & \times & 0 & \times & \mathbb{R}_9 & \times & \mathbb{C}_{z_3}
\end{array}
$$

Theory topological along $w_{1,2}$ and holomorphic along z's

Next step: Introduce $\mathcal{N} = (0,4)$ boundary conditions:

Twisted IIB supergravity:

Dirichlet boundary conditions via additional D5:

bulk:	$\mathbb{C}_{z_1}$	$\times$	$\mathbb{C}_{w_2}$	$\times$	$\mathbb{C}_{z_2}$	$\times$	$\mathbb{C}_{w_1}$	$\times$	$\mathbb{C}_{z_3}$
	$\cong$		$\cong$		$\cong$		$\cong$		$\cong$
	$\mathbb{R}^2_{01}$	$\times$	$\mathbb{R}^2_{26}$	$\times$	$\mathbb{R}^2_{34}$	$\times$	$\mathbb{R}^2_{59}$	$\times$	$\mathbb{R}^2_{78}$
D3:	$\mathbb{C}_{z_1}$	$\times$	$(\mathbb{R}_+)_2 \times I_6$	$\times$	0	$\times$	0	$\times$	$0.$
D5:	$\mathbb{C}_{z_1}$	$\times$	$\mathbb{R}_2 \times \partial I_6$	$\times$	0	$\times$	$\mathbb{R}_9$	$\times$	$\mathbb{C}_{z_3}$
D5′:	$\mathbb{C}_{z_1}$	$\times$	$0 \times \mathbb{R}_6$	$\times$	$\mathbb{C}_{z_2}$	$\times$	$\mathbb{R}_5$	$\times$	0

- Which are further admissible deformations?

  Look among **residual** supersymmetries and control which is compatible with the configuration:

Twisted IIB supergravity:

- Two possible deformations identified with odd elements of $3d\ \mathcal{N}=4$:

$\mathcal{Q}_C = \partial_{z_1} \wedge \partial_{z_2}$

- Acts on the D-brane gauge theory as: $\partial_{z_1} \wedge \partial_{z_2} \to \epsilon_2 \partial_{z_1}$
- Deformation renders z_2 topological and keeps z_3 holomorphic: Coulomb branch moduli become Q-exact $\longrightarrow$ C-twist

✓ Holomorphic d.o.f on the boundary $\longrightarrow$ Compatible with Dirichlet b.c for the free hyper

$\mathcal{Q}_H = z_2$

- Acts on the D-brane gauge theory as: $z_2 \to \partial_{\epsilon_2}$
- Deformation renders z_3 topological and keeps z_2 holomorphic: Higgs branch moduli become Q-exact $\longrightarrow$ H-twist

✘ No solutions on the boundary

expected from field theory

Concluding remarks

- Construction of holomorphic boundary conditions for $3d\ \mathcal{N} = 4$ theories via holomorphic brane engineering:

  - ✓ Embedding Hanany-Witten brane configurations in IIB twisted supergravity
  - ✓ Deformation from holomorphic to topological twist
  - ✓ Results in agreement with field theory and with purely holomorphic field theory construction

- Unexplored directions

  - → Study of interacting theories
  - → Bulk defect operators in 3d and 4d (work in progress with I.Brunner and I.Saberi)
  - → Twisted Holographic configurations

Thank you for your attention…!