BOOTSTRAPPING ADS/BCFT

Caltech

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Based on [arXiv:2206.03035]
boundary

\[ T = \bar{T} \bigg|_{bdy} \]
Semiclassical gravity \( (c = \frac{3}{2G_N} \gg 1) \) with massive particles and **ETW branes**

\[
I_{grav} = -\frac{1}{16\pi G_N} \int_M d^3 x \sqrt{g} (R + 2) + \sum_i m_i \int l_i - \frac{1}{8\pi G_N} \int_Q d^2 x \sqrt{h} (K - T) \]

\( \partial (ETW) = \text{bdy. of CFT} \)
AdS/BCFT

\[ I_{\text{grav}} = -\frac{1}{16\pi G_N} \int_M d^3x \sqrt{g} (R + 2) + \sum_i m_i \int_i dl_i - \frac{1}{8\pi G_N} \int_Q d^2x \sqrt{h} (K - T) \]

Induced metric: \( h_{\mu\nu} = g_{\mu\nu} - n_{\mu} n_{\nu} \), 
Extrinsic curvature: \( K_{\mu\nu} = h_{\mu}^{\ \rho} h_{\nu}^{\ \lambda} \nabla_{\rho} n_{\lambda} \)

**Neumann b.c.** is imposed on the brane (Einstein eq. of brane). 

\[ K_{ab} - Kh_{ab} = -T h_{ab} \]
Issues in AdS/BCFT

BCFT

AdS with ETW brane
Issues in AdS/BCFT

Massive particle produces deficit angle

\[ \delta \theta = 2\pi \left( 1 - \sqrt{1 - \frac{c}{24} h_i} \right) \]

Pointed out by
[Geng, Lust, Mishra, Wakeham]
[Kawamoto, Mori, Suzuki, Takayanagi]
[Bianchi, De Angelis, Meineri]

The first one proposed that \( h_i \in \left[ \frac{c}{32}, \frac{c}{24} \right) \) should be excluded in holographic CFT

Self-intersection?
**Issues in AdS/BCFT**

\[ h_i = 0 \]
\[ 0 < h_i < \frac{c}{32} \]
\[ \frac{c}{32} < h_i \]

Comment:
Bottom-up construction is very naïve.
Not so surprising if something wrong happens.

Actually, this bottom-up model still works as we will show.
Goal

Our goal is to give a CFT understanding of:

- intersection

- self-intersection

and an understanding of Island/BCFT correspondence
Review of BCFT

New ingredient (boundary primary)

Primary operator living on boundary, which can change boundary condition. Same transformation law under conformal mapping.
**Review of BCFT**

state – operator like mapping

Conformal weight of $\phi^a_b$

= Energy corresponding to the state on the strip
Review of BCFT [Lewellen]

Cutting: Inserting (bulk operator) complete set
Review of BCFT

\[ \sum_p C_{p0} C_{ijp} \mathcal{F}_{ji}^j (p|z) \]

\( \mathcal{F}_{ji}^j \) is fixed by conformal sym. & mirror method

[Lewellen]
Review of BCFT

\[
\sum_{p} C_{p0} C_{ijp} \mathcal{F}_{ji}^{ii} (p|z)
\]

Note:

\[\mathcal{F}_{ji}^{ii} (p|z) = \text{Virasoro block.}\]

Because Ward id (with bdy) is equivalent to Ward id (without bdy) by mirror method

kinematic part = conformal block
Review of BCFT

\[ \sum_p C_{p0} C_{ijp} \mathcal{F}^{ji}_{ji}(p|z) \]

Note:
\[ \mathcal{F}^{ji}_{ji}(p|z) = \text{Virasoro block.} \]

Because Ward id (with bdy) is equivalent to Ward id (without bdy) by mirror method

\[ \sum_{p,\bar{p},N,\bar{N}} \langle \phi_i | \phi_j | L_N \phi_p \rangle \langle \phi_i | \phi_j | L_{\bar{N}} \phi_{\bar{p}} \rangle \langle L_{-N} L_{-\bar{N}} \phi_{p,\bar{p}} \rangle_{disk} \]

\[ = \sum_{p,\bar{p},N,\bar{N}} \langle \phi_i | \phi_j | L_N \phi_p \rangle \langle \phi_{\bar{i}} | \phi_{\bar{j}} | L_{\bar{N}} \phi_{\bar{p}} \rangle \langle L_{-N} \phi_p | L_{-\bar{N}} \phi_{\bar{p}} \rangle \]

\[ = \sum_{p,N} \langle \phi_i | \phi_j | L_N \phi_p \rangle \langle \phi_{\bar{i}} | \phi_{\bar{j}} | L_{-N} \phi_p \rangle \]
or equivalently, using bulk-boundary OPE
\[ \phi_i(z) \sim \sum_P C_{iP} (2\Re z)^{h_P - h_i - \bar{h}_i} \phi_P(\Re z) + \ldots \]

**Cutting:**
Inserting *(boundary)* operator complete set
Review of BCFT

\[ P_i P_j = \sum_p C_{ip} C_{jp} \mathcal{F}_{jj}^{ji}(p|z) \]

bulk-boundary OPE coef.
Bootstrap

\[ \sum_{p} C_{ip}^2 |F_{ii}(p|z)|^2 = \sum_{q} C_{iq}^2 |F_{ii}(q|1-z)|^2 \]

\[ \rightarrow \text{constraints on CFT data} \]
Analytic Bootstrap

\[ p = \sum_{q} C_{i \bar{u} q}^2 |F_{i i}^q (q | 1 - z)|^2 \]
Analytic Bootstrap

\[ \sim \mathcal{F}_{ii}^{ii}(0|1 - z) \]

bootstrap

vacuum block approximation by \( z, \bar{z} \to 0 \) (Cardy formula)
\( \bar{z} \to 0 \) (large-spin)
Analytic Bootstrap

$C_{iip}^2 \simeq F_{0q} \left[ \begin{array}{c} i \\ i \\ i \end{array} \right]$
Boundary Averaging

- state/operator-like correspondence

\[ |B^a\rangle = g^a \sum_p C^a_{p\|} |p\rangle \] 

**Assumption:**

\[ \overline{C^a_{p\|}} = \delta_{p\|} \]

suggested by

- no interaction with brane
  - [Takayanagi], [Fujita, Takayanagi, Tonni], [Suzuki, Takayanagi]
- island model
  - [Suzuki, Takayanagi]

→ It is worth investigating this condition by bootstrap.
Our interest is ADM mass

graviton interaction

boundary

ETW brane
Bootstrap

\[ \text{ADM mass} = C_{p0} C_{ii} p F_{ii} (p | 1 - z) \]
Bootstrap

By assumption
Bootstrap

ADM mass

By assumption

\[ P = i \]

Liouville momentum

\[ c = 1 + 6Q^2, \quad h_i = \alpha_i (Q - \alpha_i) \]

By assumption

\[ F_{ii}^{ii} (0|1 - z) = \int d\alpha_P F_{0P} \left[ \begin{array}{c} i \\ i \end{array} \right] F_{ii}^{ii} (P|z) \]
Bootstrap

ADM mass = lowest primary dimension

\[ \alpha_P = 2\alpha_i \]

By assumption

\[ F_{ii}^{ii}(0|1-z) = \int d\alpha_P F_{0P} \left[ \begin{array}{c} i \\ i \end{array} \right] F_{ii}^{ii}(P|z) \]
Implication

ADM mass = lowest primary dimension

\[ \alpha_P = 2\alpha_i \]

self-intersection bound

\[ h_i \leq \frac{c}{32} \iff h_P \leq \frac{c}{24} \]

→ self-intersection can be avoided by blackhole formation
Comments

- Relation to light-cone bootstrap

\[ h_i \geq \frac{c}{32} \]

- Relation to island model

\[ O_i = O_i = 0 \]

\[ O_i \times O_i \neq 0 \rightarrow \text{replica wormholes?} \]
Comments

- ADM mass from gravity side

  ADM mass calculation on gravity side is complicated, but we show an exact match between CFT calculation & gravity calculation in [YK, Wei] [Kawamoto, Mori, Suzuki, Takayanagi, Ugajin]

Note:
Averaging sometimes simplifies calculations. In this sense, averaging is thought of as a useful tool to evaluate something in gravity.
Discussion

- Explicit example?
  One simple realization may be obtained in averaged Narain CFT

- Wormholes in BCFT? [under consideration]
  Averaging simplifies evaluation of classical saddle.
  One may obtain a new understating of wormholes in braneworld

- Bootstrapping AdS/BCFT [YK, Wei]
  Many loopholes in AdS/BCFT
  Bootstrap can give correct formulation of AdS/BCFT

- Detailed connection to island model?