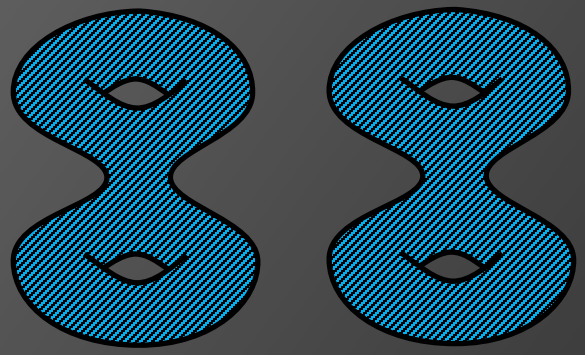
BOOTSTRAPPING ADS/BCFT

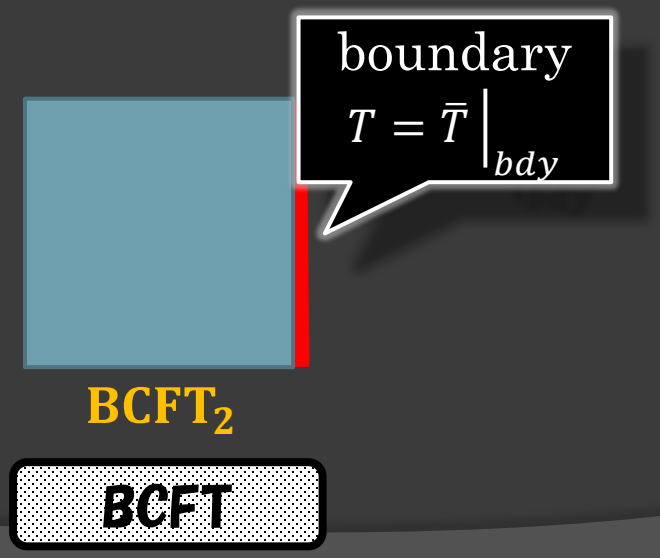


Caltech

Yuya Kusuki

Based on [arXiv:2206.03035]

BCFT [Cardy]

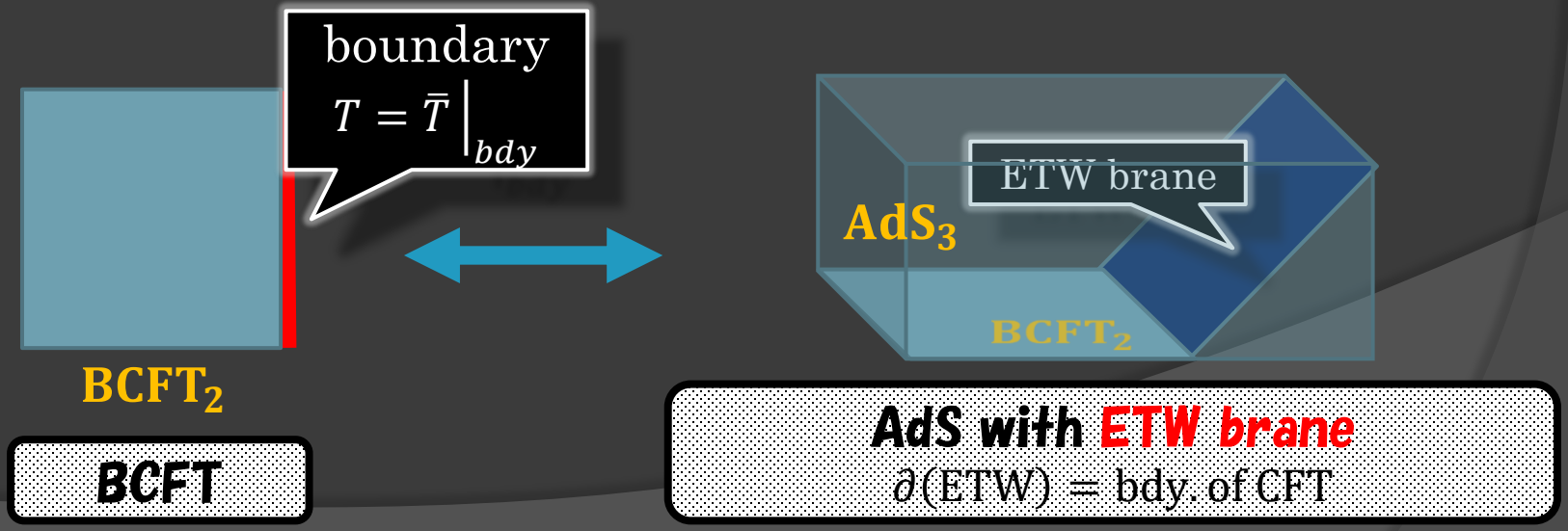


AdS/BCFT

[Takayanagi]
[Fujita, Takayanagi, Tonni]

$$I_{grav} = -\frac{1}{16\pi G_N}\int_M d^3x\,\sqrt{g}(R+2) + \sum_i m_i \int dl_i - \frac{1}{8\pi G_N}\int_Q d^2x\,\sqrt{h}(K-T)$$

Semiclassical gravity ($c = \frac{3}{2G_N} \gg 1$) with massive particles and **ETW branes**



BCFT

AdS with ETW brane
∂(ETW) = bdy. of CFT

AdS/BCFT

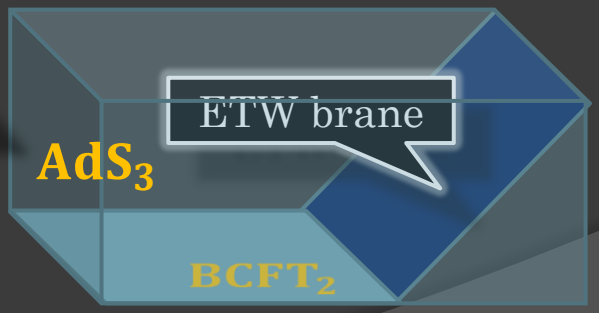
[Takayanagi]
[Fujita, Takayanagi, Tonni]

$$I_{grav} = -\frac{1}{16\pi G_N}\int_M d^3x\sqrt{g}(R+2) + \sum_i m_i \int dl_i - \frac{1}{8\pi G_N}\int_Q d^2x\sqrt{h}(K-T)$$

Induced metric: $h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$,
Extrinsic curvature: $K_{\mu\nu} = h_\mu{}^\rho h_\nu{}^\lambda \nabla_\rho n_\lambda$

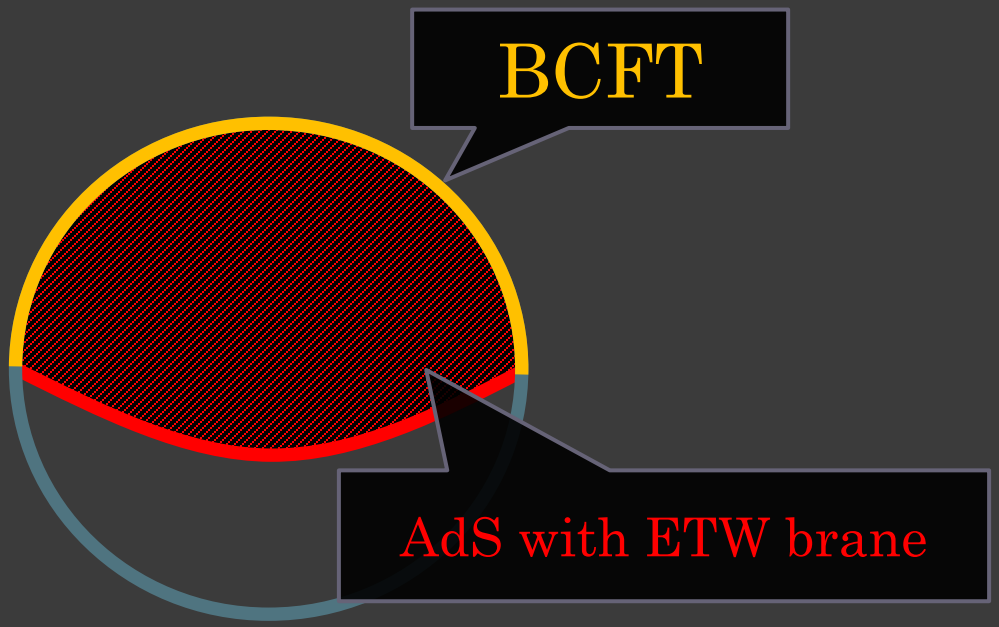
Neumann b.c. is imposed on the brane (Einstein eq. of brane).

$$K_{ab} - Kh_{ab} = -Th_{ab}$$

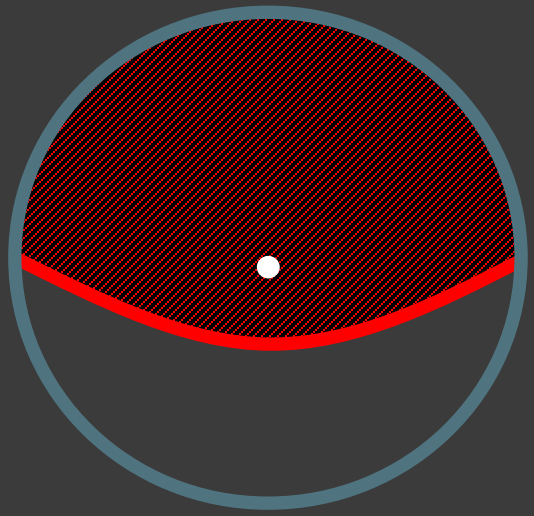


AdS with ETW brane
∂(ETW) = bdy. of CFT

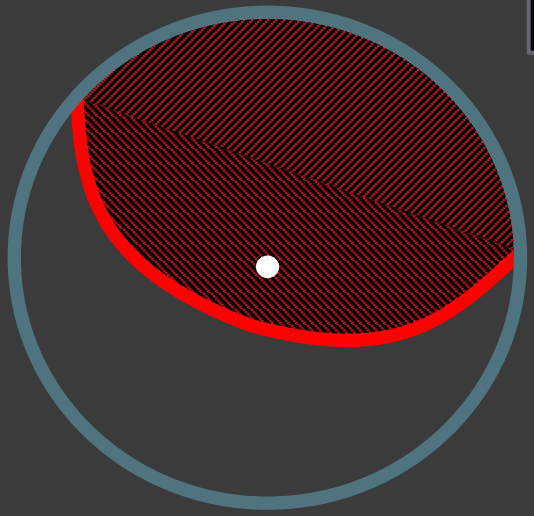
Issues in AdS/BCFT



Issues in AdS/BCFT

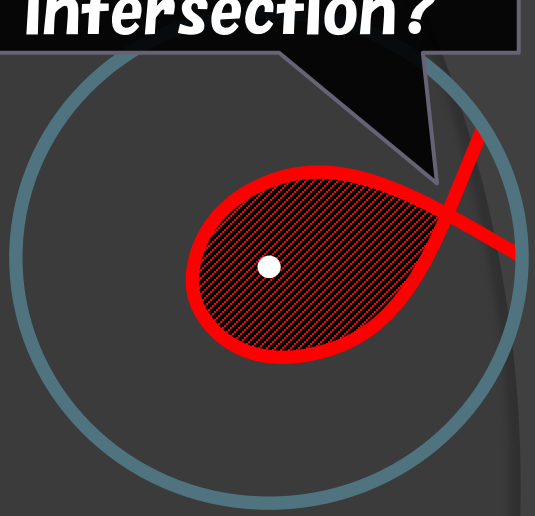


$h_i = 0$



$0 < h_i < \frac{c}{32}$

Self-intersection?



$\frac{c}{32} < h_i$

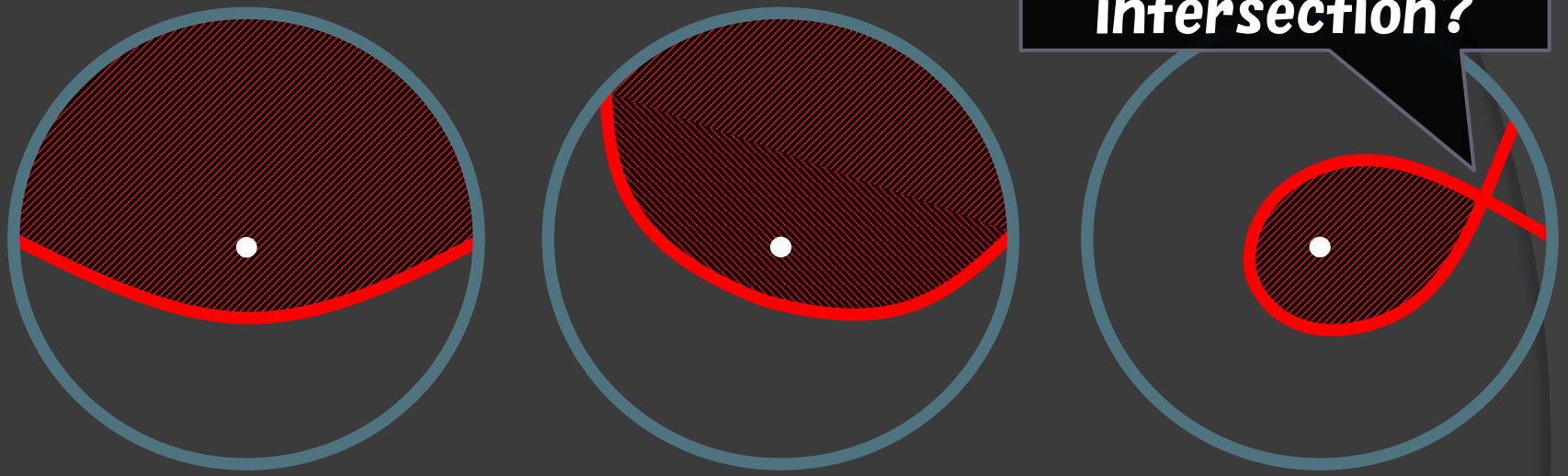
Massive particle produces deficit angle

$$\delta\theta = 2\pi\left(1 - \sqrt{1 - \frac{c}{24}h_i}\right)$$

Pointed out by
[Geng, Lust, Mishra, Wakeham]
[Kawamoto, Mori, Suzuki, Takayanagi]
[Bianchi, De Angelis, Meineri]

The first one proposed that $h_i \in \left[\frac{c}{32}, \frac{c}{24}\right)$ should be excluded in holographic CFT

Issues in AdS/BCFT



$h_i = 0$ $\quad 0 < h_i < \frac{c}{32}$ $\quad \frac{c}{32} < h_i$

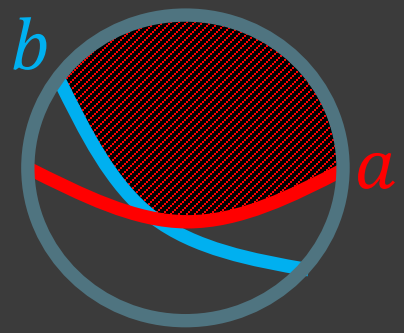
Comment:
Bottom-up construction is very naïve.
Not so surprising if something wrong happens.

Actually, this bottom-up model still works as we will show.

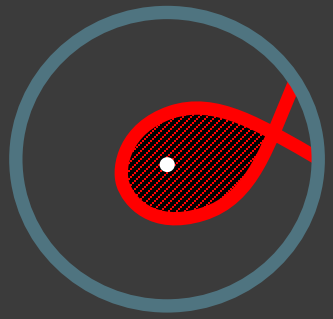
Goal

Our goal is to give a **CFT understanding** of

- **intersection**



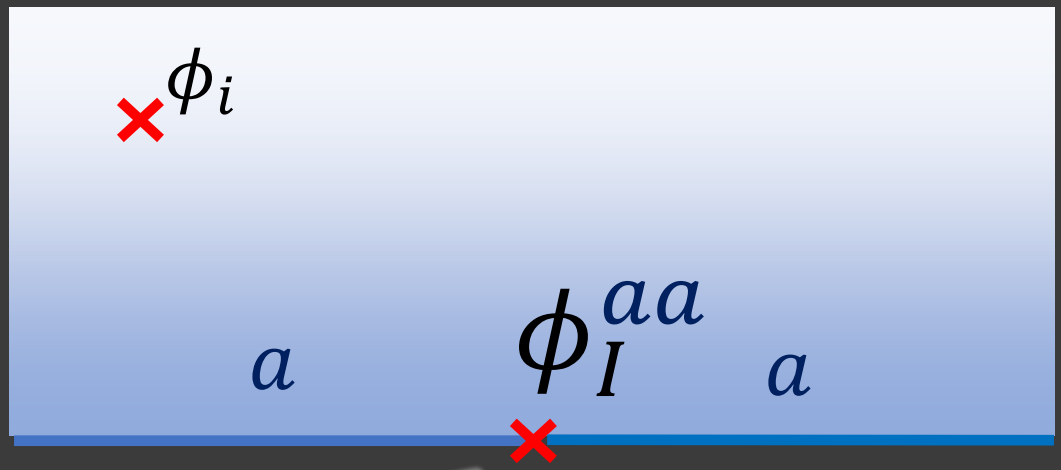
- **self-intersection**



and an understanding of Island/BCFT correspondence

Review of BCFT

i: bulk
I: boundary

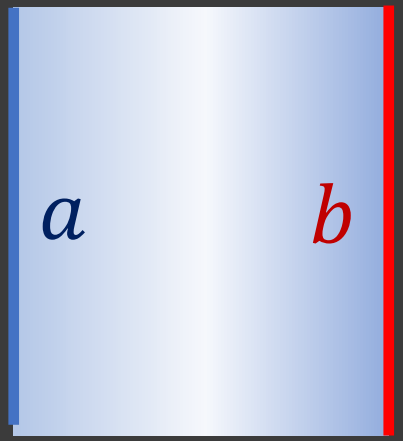


New ingredient (boundary primary)

Primary operator living on boundary,
which can change boundary condition.
Same transformation law under conformal mapping.

Review of BCFT

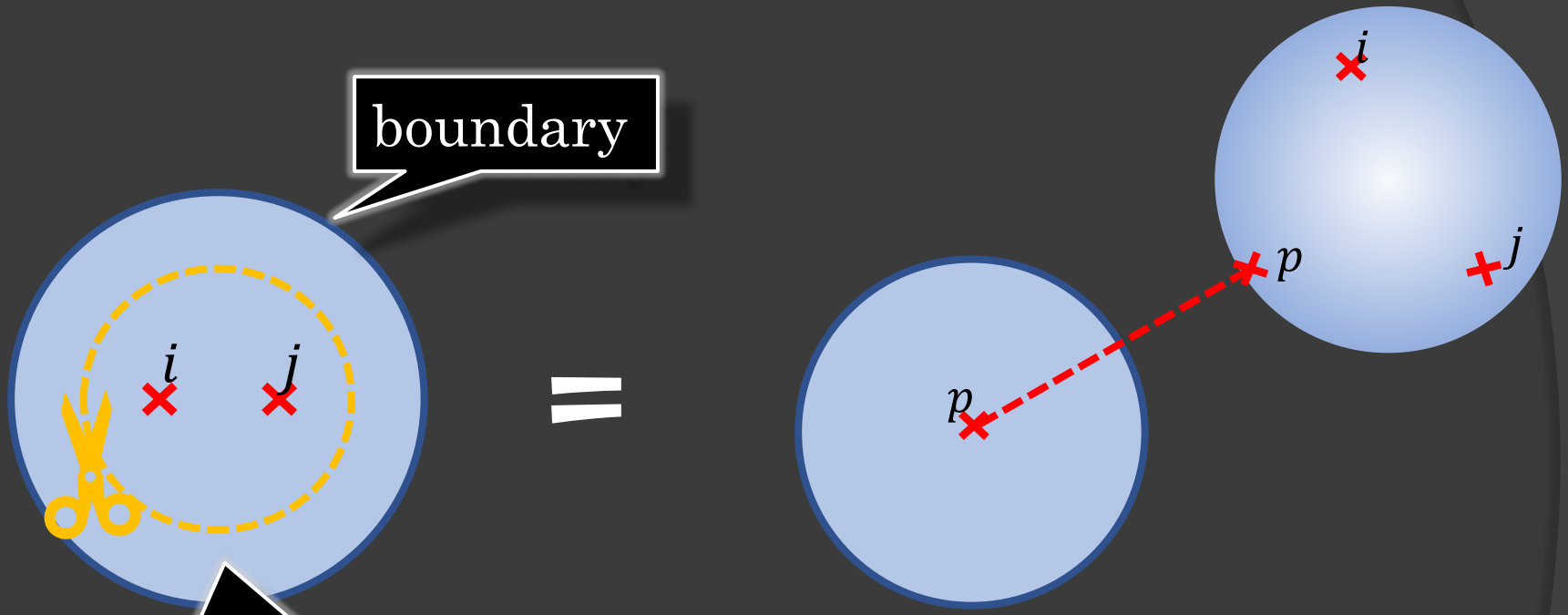
state – operator like mapping



Conformal weight of ϕ_I^{ab}

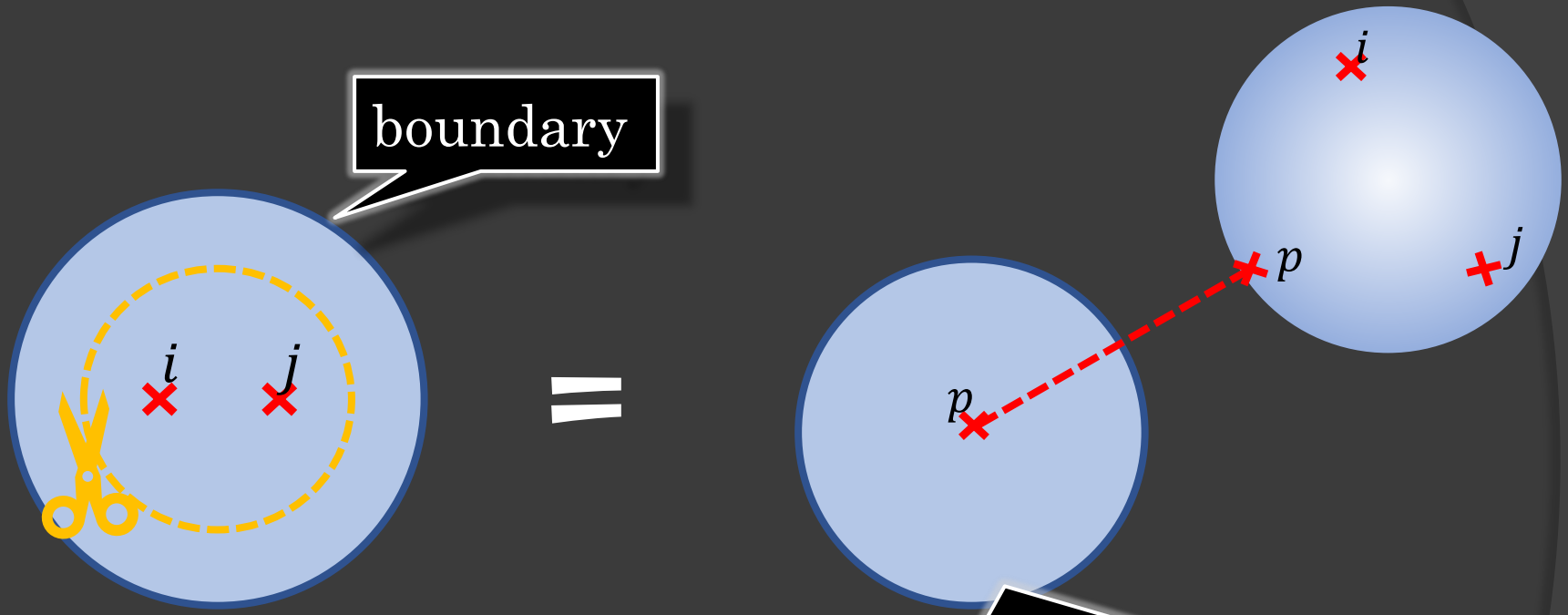
= Energy corresponding to the state on the strip

Review of BCFT [Lewellen]



Cutting:
Inserting (bulk operator) complete set

Review of BCFT [Lewellen]

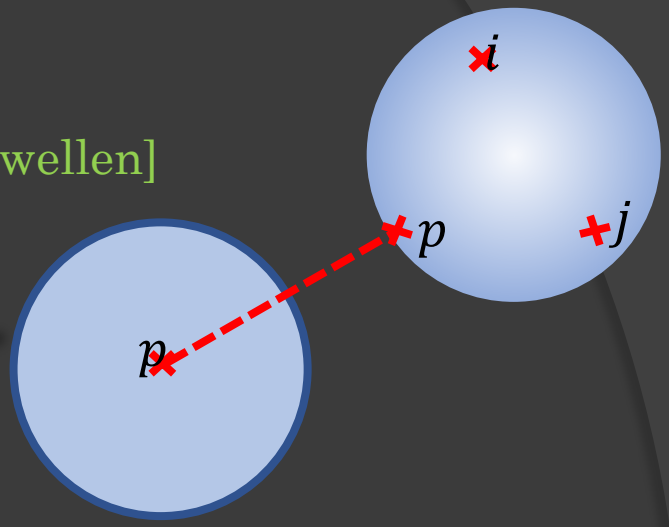


$$\sum_p C_{p0} C_{ijp} \mathcal{F}_{\overline{j}\overline{i}}^{ji}(p|z)$$

$\mathcal{F}_{\overline{j}\overline{i}}^{ji}$ is fixed by conformal sym. & mirror method

Review of BCFT [Lewellen]

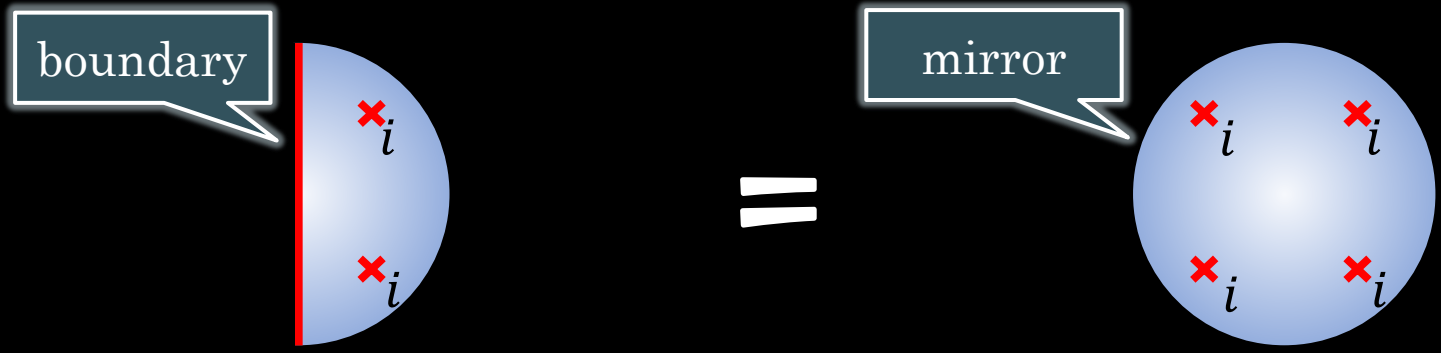
$$\sum_p C_{p0} C_{ijp} \mathcal{F}_{\overline{j}\overline{i}}^{ji}(p|z)$$



Note:

$\mathcal{F}_{\overline{j}\overline{i}}^{ji}(p|z)$ = Virasoro block.

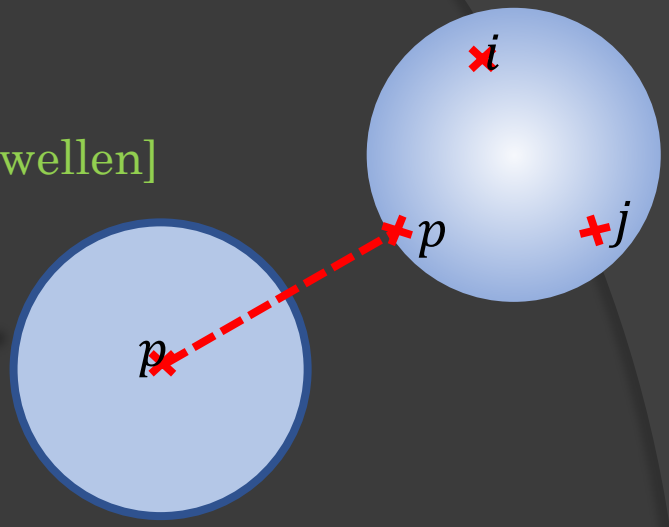
Because Ward id (with bdy) is equivalent to Ward id (without bdy) by mirror method



kinematic part = conformal block

Review of BCFT [Lewellen]

$$\sum_p C_{p0} C_{ijp} \mathcal{F}_{\bar{j}\bar{i}}^{ji}(p|z)$$



Note:

$\mathcal{F}_{\bar{j}\bar{i}}^{ji}(p|z)$ = Virasoro block.

Because Ward id (with bdy) is equivalent to Ward id (without bdy) by mirror method

$$\sum_{p,\bar{p},N,\bar{N}} \langle \phi_i | \phi_j | L_{-N}\phi_p \rangle \langle \phi_{\bar{\imath}} | \phi_{\bar{\jmath}} | L_{-\bar{N}}\phi_{\bar{p}} \rangle \langle L_{-N}L_{-\bar{N}}\phi_{p,\bar{p}} \rangle_{disk}$$

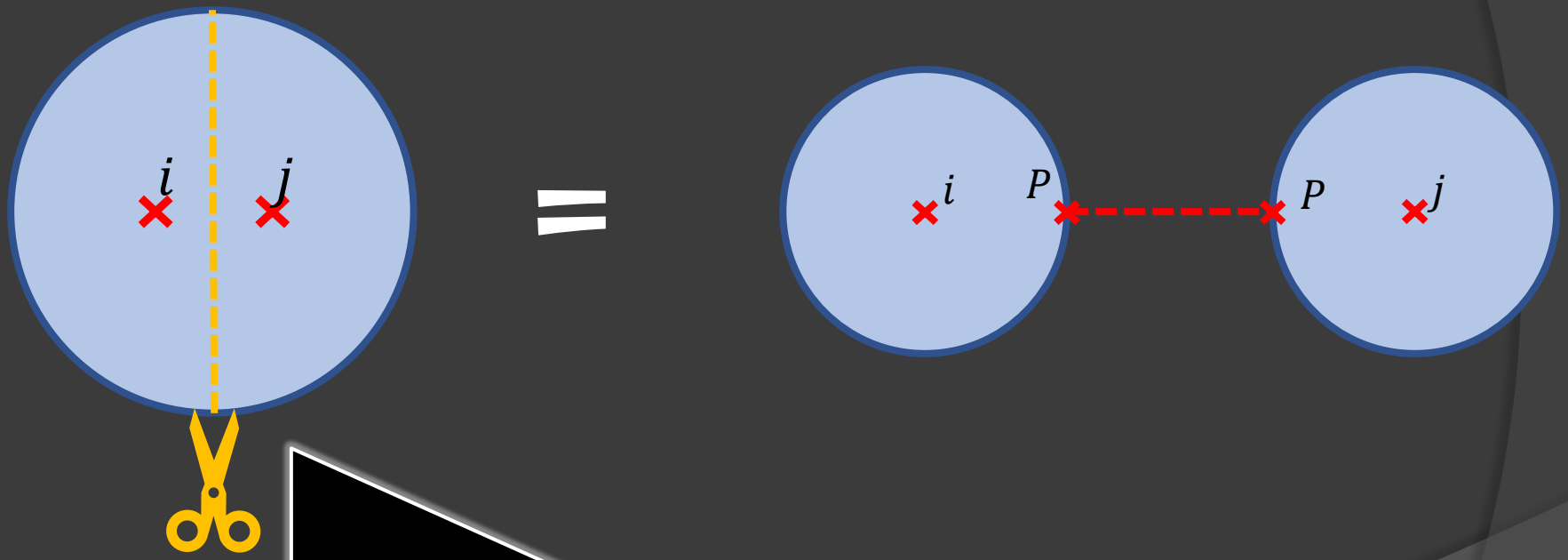
$$= \sum_{p,\bar{p},N,\bar{N}} \langle \phi_i | \phi_j | L_{-N}\phi_p \rangle \langle \phi_{\bar{\imath}} | \phi_{\bar{\jmath}} | L_{-\bar{N}}\phi_{\bar{p}} \rangle \langle L_{-N}\phi_p \, | L_{-\bar{N}}\phi_{\bar{p}} \rangle$$

$$= \sum_{p,N} \langle \phi_i | \phi_j | L_{-N}\phi_p \rangle \langle \phi_{\bar{\imath}} | \phi_{\bar{\jmath}} | L_{-N}\phi_p \rangle$$

Review

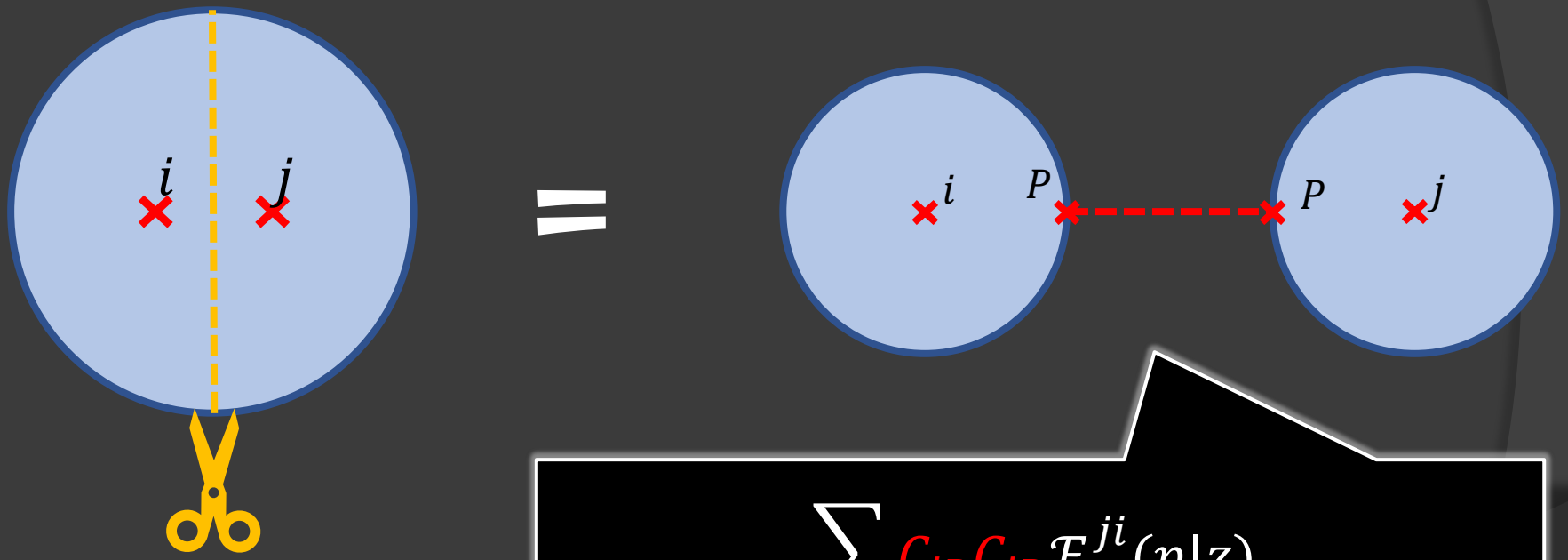
or equivalently, using bulk-boundary OPE

$$\phi_i(z) \sim \sum_P C_{iP}(2\Im z)^{h_P - h_i - \bar{h}_i}\phi_P(\Re z) + \cdots$$



Cutting:
Inserting (**boundary** operator) complete set

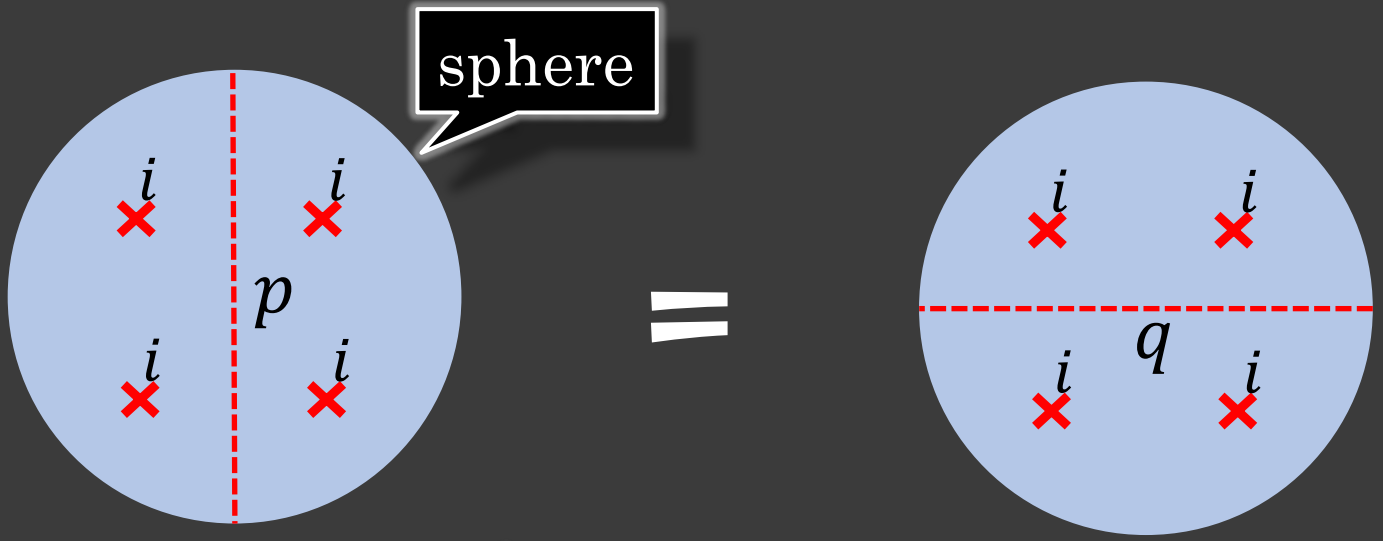
Review of BCFT [Lewellen]



$$\sum_p C_{iP} C_{jP} \mathcal{F}_{\overline{j}\overline{i}}^{ji}(p|z)$$

bulk-boundary OPE coef.

Bootstrap

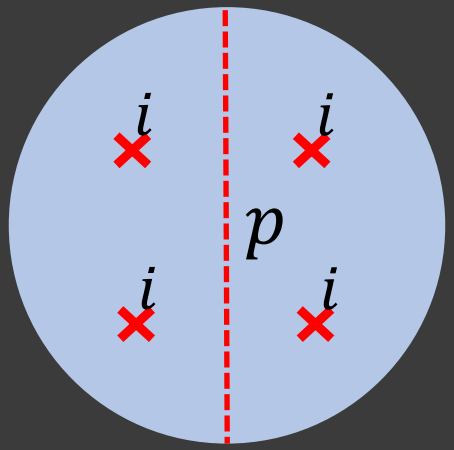


$$\sum_p C_{iip}^2 \left|\mathcal{F}_{ii}^{ii}(p|z)\right|^2 = \sum_q C_{iiq}^2 \left|\mathcal{F}_{ii}^{ii}(q|1-z)\right|^2$$

→ constraints on CFT data

Analytic Bootstrap

bootstrap



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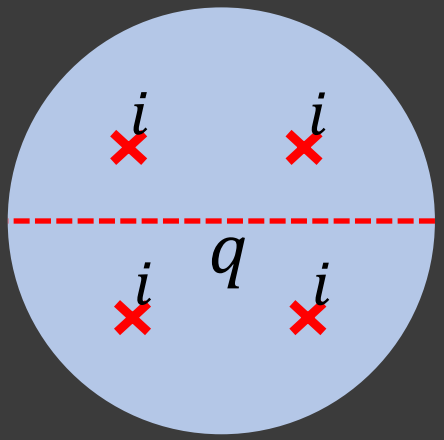
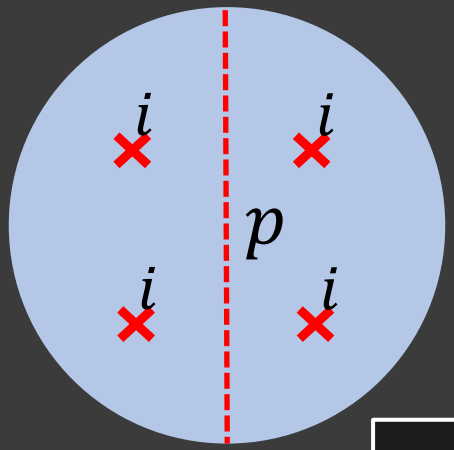


$$= \sum_q C_{iiq}^2 \left| \mathcal{F}_{ii}^{ii}(q|1-z) \right|^2$$

Analytic Bootstrap

bootstrap



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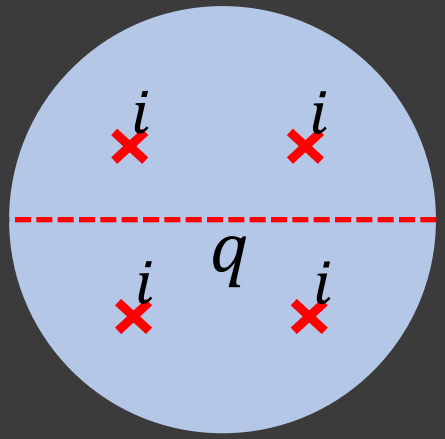


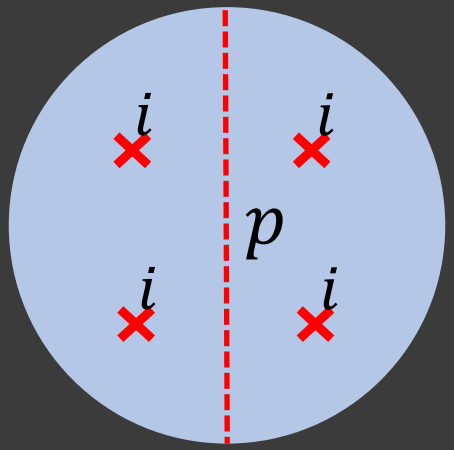
$\simeq \mathcal{F}_{ii}^{ii}(0|1-z)$

vacuum block approximation by
$z, \bar{z} \to 0$ (Cardy formula)
$\bar{z} \to 0$ (large-spin)

Analytic Bootstrap

[YK]
[Collier, Gobeil, Maxfield, Perlmutter]

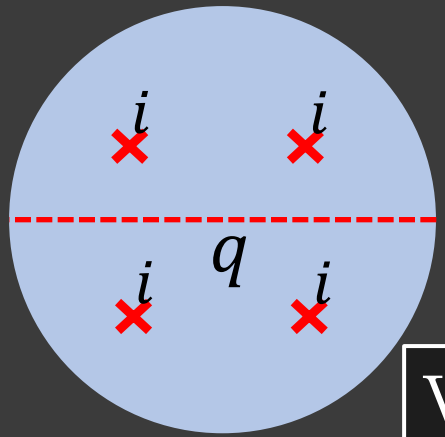
bootstrap



$$C_{iip}^2 \simeq F_{0q}\begin{bmatrix} i & i \\ i & i \end{bmatrix}$$

=

=



$$\simeq \mathcal{F}_{ii}^{ii}(0|1-z) = \int \mathrm{d}\alpha_q \, F_{0q}\begin{bmatrix} i & i \\ i & i \end{bmatrix} \mathcal{F}_{ii}^{ii}(q|z)$$

Vacuum block approximation

Fusion transformation

Boundary Averaging

boundary is characterized by $C^a_{p\mathbb{I}}$

- state/operator-like correspondence

$$|B^a\rangle = g^a \sum_p C^a_{p\mathbb{I}} \, |p\rangle\rangle$$

Assumption:

$$\overline{C^a_{p\mathbb{I}}} = \delta_{p\mathbb{I}}$$

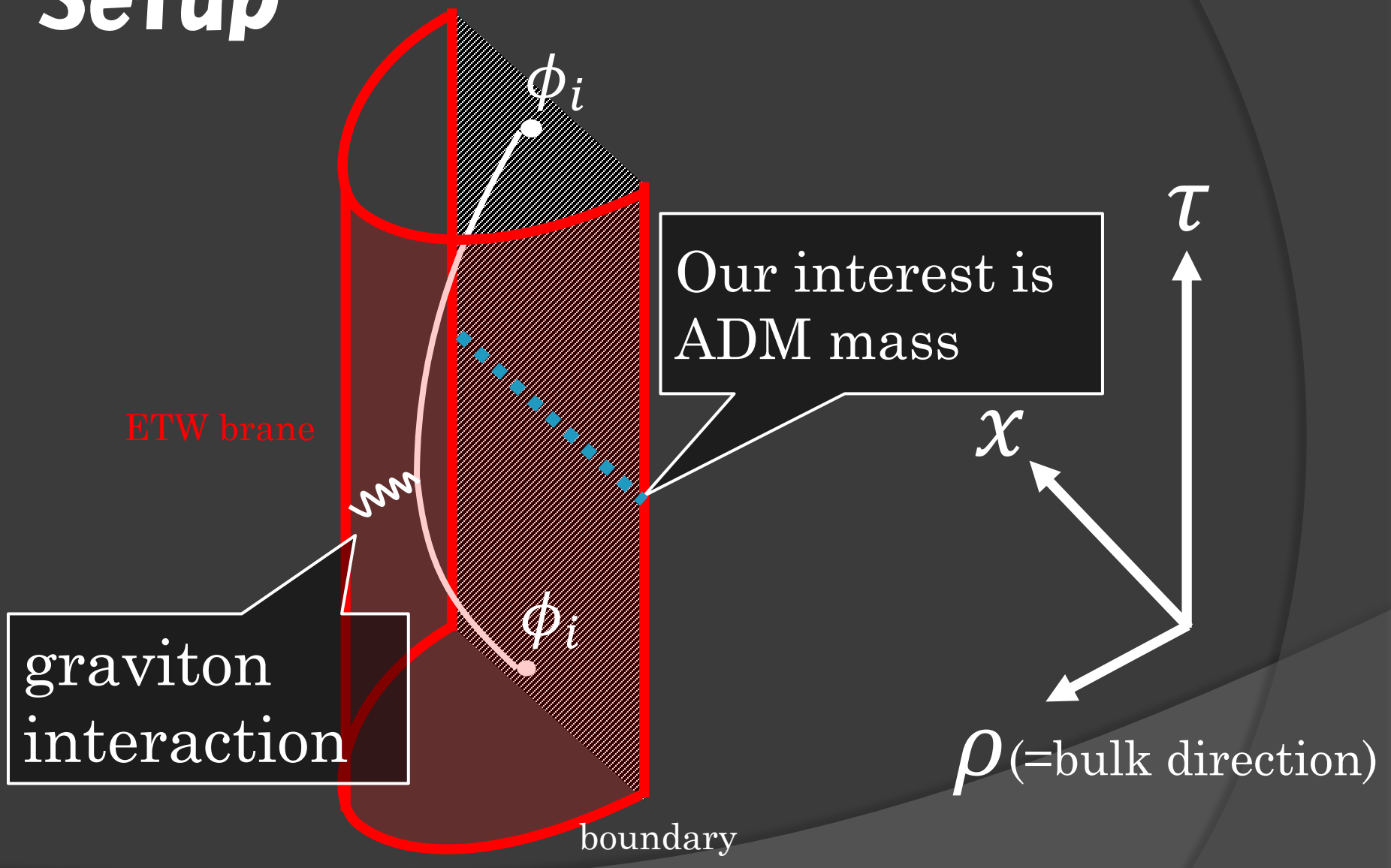
Note: $\overline{C^a_{p\mathbb{I}}} \neq 0$ will be considered in [YK, Zixia]

suggested by

- no interaction with brane
  [Takayanagi], [Fujita, Takayanagi, Tonni], [Suzuki, Takayanagi]
- island model
  [Suzuki, Takayanagi]

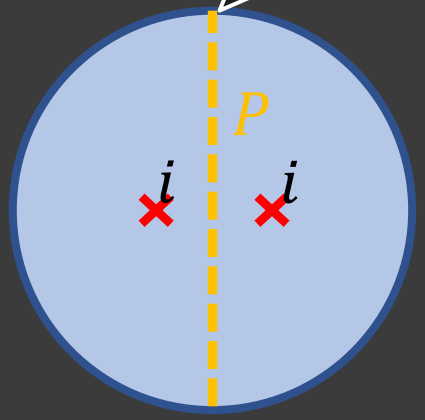
→ It is worth investigating this condition by bootstrap.

Setup


ϕ_i
Our interest is ADM mass
ETW brane
graviton interaction
ϕ_i
boundary
τ
x
ρ (=bulk direction)

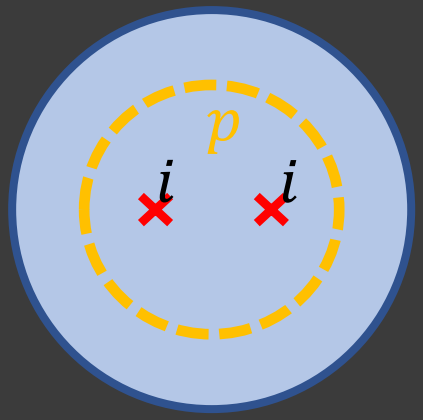
Bootstrap

ADM mass



bootstrap

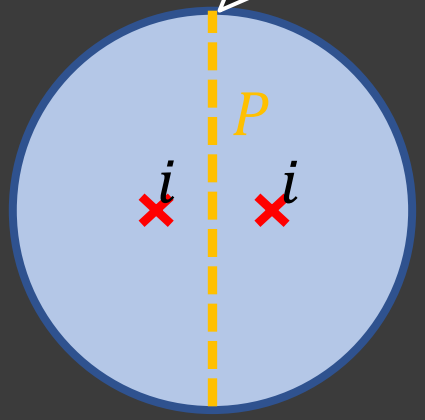
=



$$= \sum_{p} C_{p0} C_{iip} \mathcal{F}_{ii}^{ii}(p|1-z)$$

Bootstrap

ADM mass



bootstrap

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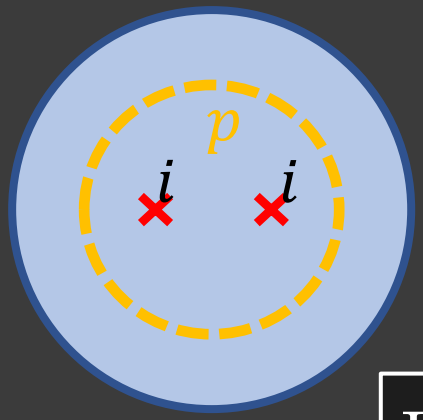
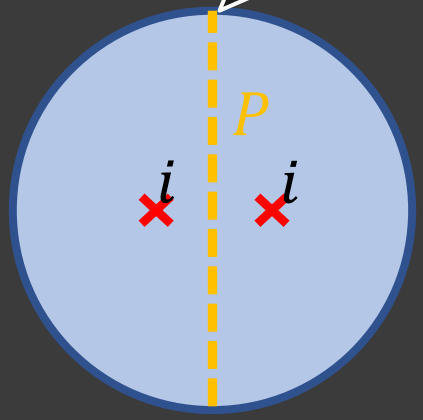


$= \mathcal{F}_{ii}^{ii}(0|1-z)$

By assumption

Bootstrap

ADM mass



bootstrap

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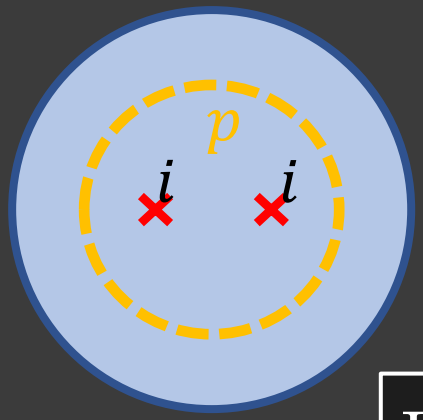


Liouville momentum

$c = 1 + 6Q^2,$
$h_i = \alpha_i(Q - \alpha_i)$

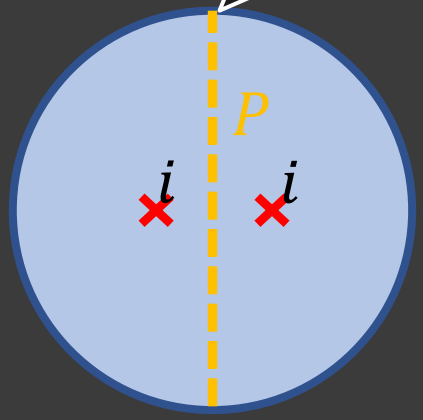
$$= \mathcal{F}_{ii}^{ii}(0|1-z) = \int \mathrm{d}\alpha_P \, F_{0P}\begin{bmatrix} i & i \\ i & i \end{bmatrix} \mathcal{F}_{ii}^{ii}(P|z)$$

By assumption

Fusion transformation

Bootstrap

ADM mass

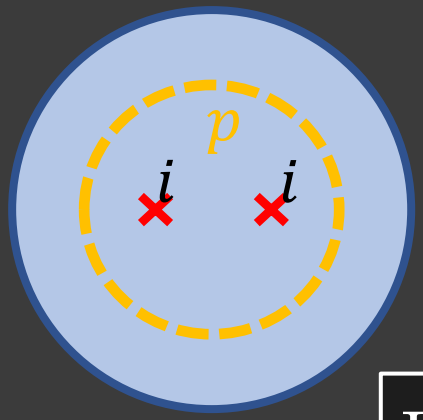


ADM mass = lowest primary dimension

$$\alpha_P = 2\alpha_i$$

bootstrap

=



=

$$\mathcal{F}_{ii}^{ii}(0|1-z) = \int \mathrm{d}\alpha_P \, F_{0P}\begin{bmatrix} i & i \\ i & i \end{bmatrix} \mathcal{F}_{ii}^{ii}(P|z)$$

By assumption

Fusion transformation

Implication

$c = 1 + 6Q^2$,
$h_i = \alpha_i(Q - \alpha_i)$

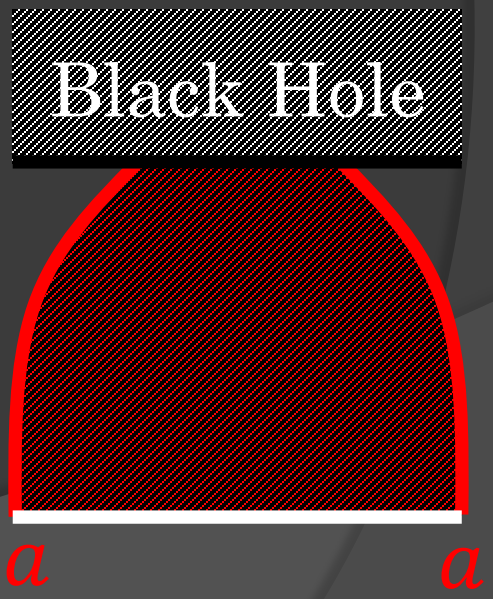
ADM mass = lowest primary dimension

$$\alpha_P = 2\alpha_i$$

self-intersection bound

$$h_i \leq \frac{c}{32} \quad \Leftrightarrow \quad h_P \leq \frac{c}{24}$$

→ self-intersection can be avoided by **blackhole formation**

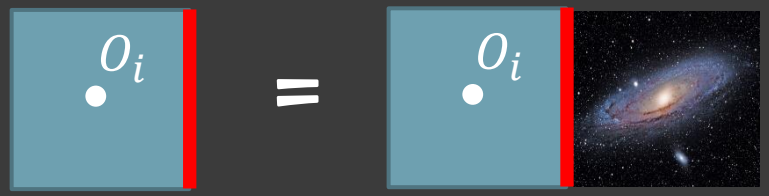


Comments

Same vacuum block approximation of bootstrap

- Relation to light-cone bootstrap

- Relation to island model



→ replica wormholes?

Comments

- ADM mass from gravity side

  ADM mass calculation on gravity side is complicated, but we show an exact match between CFT calculation & gravity calculation in [YK, Wei] [Kawamoto, Mori, Suzuki, Takayanagi, Ugajin]

Note:
Averaging sometimes simplifies calculations.
In this sense, averaging is thought of as a useful tool to evaluate something in gravity.

Discussion

- Explicit example ?

  One simple realization may be obtained in averaged Narain CFT

- Wormholes in BCFT ? [under consideration]

  Averaging simplifies evaluation of classical saddle.

  One may obtain a new understating of wormholes in braneworld

- Bootstrapping AdS/BCFT [YK, Wei]

  Many loopholes in AdS/BCFT

  Bootstrap can give correct formulation of AdS/BCFT

- Detailed connection to island model ?