Perturbative calculations in twisted 4d gauge theories


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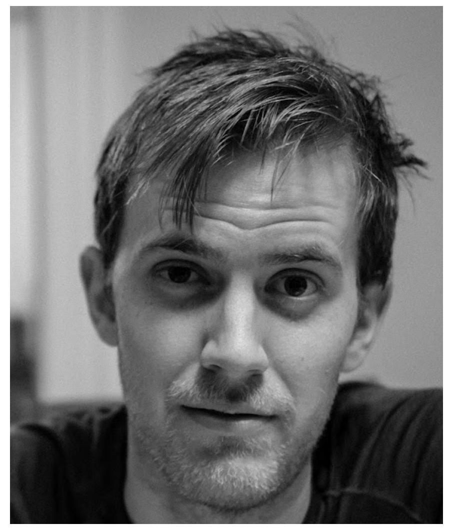
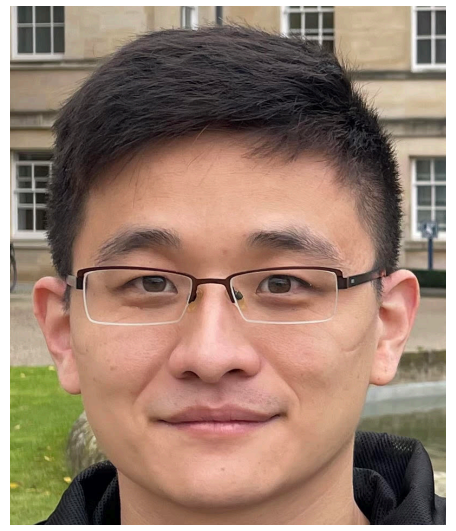

Brian Williams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4d N=1 gauge theory

- Minimal SUSY in 4d:

$$Q_{\dot{\alpha}} \in S_+ \qquad \bar{Q}_{\alpha} \in S_- \qquad\qquad \{Q_{\dot{\alpha}}, \bar{Q}_{\beta}\} = P_{\dot{\alpha}\beta}$$

- Interesting RG physics: Confinement, chiral symmetry breaking, etc.
- (Seiberg) dualities, holography (IIB $AdS_5 \times Y^5$, D3 @ CY cone)
- Growing mathematical connections

Holomorphic twist

- Pass to cohomology of $Q = Q_{\dot{-}}$ (add to BRST charge)
  - Simplified theory, mathematically more treatable
  - Holomorphic: $Q_{\dot{-}}\mathcal{O} = 0 \;\; \rightarrow \;\; \partial_{\dot{-}\alpha}\mathcal{O} = Q_{\dot{-}}(\bar{Q}_\alpha \mathcal{O})$
- Infinite (derived) symmetry algebra **Gwilliam, Saberi, Williams**
  - Complex symplectomorphisms of space-time
  - Holomorphic flavour rotations / 4d Kac-Moody

Traditional SUSY tools I

- N=1 SCFT: SU(2,2|1) superconformal symmetry
- Superconformal index: $\mathrm{Tr}(-1)^F p^{j_1+j_2-r/2} q^{j_1-j_2-r/2} e^{-\beta\{Q_{\dot{-}}, S^{\dot{-}}\}}$
  - Witten index, counts operators annihilated by $Q_{\dot{-}}, S^{\dot{-}}$
  - Computable from free theory
- Invariant under dualities (elliptic hypergeometric identities)
- Same as cohomology of $Q = Q_{\dot{-}}$: (holomorphic) twist!
  - Partition function on $(\mathbb{C}^2 - (0,0))/\{z_1 \to p z_1, z_2 \to q z_2\}$

Traditional SUSY tools II

- Chiral operators: $Q_{\dot{\alpha}}\mathcal{O} = 0$ modulo $Q_{\dot{\alpha}}\mathcal{O}'$
  - Almost a cohomology, but not quite
  - $\partial_{\dot{\alpha}\beta}\mathcal{O} = Q_{\dot{\alpha}}(\bar{Q}_{\beta}\mathcal{O})$ ==> Non-singular OPE: chiral ring
    - SUSY vacua ==> spectrum of chiral ring
  - Constrained quantum corrections
- F-terms: $\int d^4x(\bar{Q}_1\bar{Q}_2\mathcal{O})$

Local operators

- Semi-chiral ring: no meromorphic OPE singularities
  - Descent relation: $\mathcal{O}^{(0)} \to \mathcal{O} = e^{d\bar{z}^\alpha \bar{Q}_\alpha} \mathcal{O}^{(0)}$ $\qquad (Q + \bar{\partial})\mathcal{O} = 0$
- Lambda bracket: $\{A, B\}_\lambda = \int_{|z|=\epsilon} A(z, \bar{z}) B(0) e^{\lambda \cdot z} d^2 z$
- Higher operations: $\int_\gamma \left[\prod_i A_i(z_i, \bar{z}_i) \right] \rho(z_*, \bar{z}_*) B(0)$
- Holomorphic factorization algebra

Special Symmetries

- SCFT: Stress tensor S_α $\{S_\alpha, \mathcal{O}\}_\lambda = \mathcal{L}_{e^{\lambda \cdot z}\partial_{z^\alpha}} \mathcal{O}$
  - Extends holomorphic conformal group
- Non-SCFT: $QS_\alpha = \partial_\alpha B$ $\{\partial^\alpha S_\alpha, \mathcal{O}\}_\lambda = \mathcal{L}_{e^{\lambda \cdot z}\lambda^\alpha \partial_{z^\alpha}} \mathcal{O}$
- Flavor current J
  - Holomorphic flavour rotations

Quantum corrections

- Perturbative and non-perturbative
- Perturbative example: 1-loop beta function
  - Generalized Konishi anomaly, etc.
  - Q(tree) + Q(1-loop) not nilpotent: higher loops needed
    - Which Feynman diagrams?
- Unknown non-perturbative corrections
  - Symmetry constraints?

Perturbative corrections

- Determined by higher brackets in free theory (assume homotopy transfer)

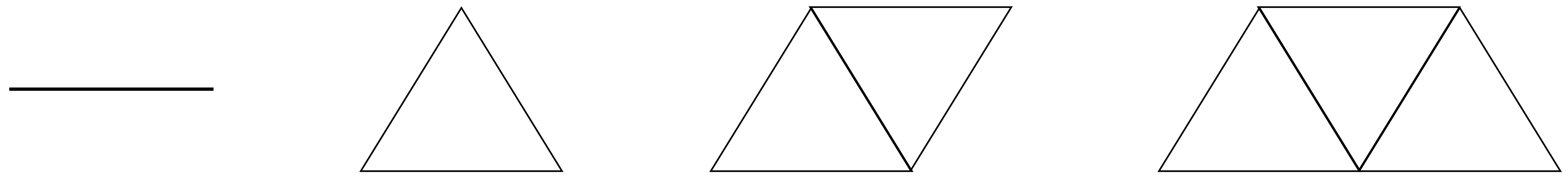
$$Q\mathcal{O} = \{\mathcal{I}, \mathcal{O}\}_0 + \{\mathcal{I}, \mathcal{I}, \mathcal{O}\}_{0,0} + \{\mathcal{I}, \mathcal{I}, \mathcal{I}, \mathcal{O}\}_{0,0,0} + \cdots$$

$$\{\mathcal{O}_1, \mathcal{O}_2\}_\lambda = \{\mathcal{O}_1, \mathcal{O}_2\}_\lambda + \{\mathcal{I}, \mathcal{O}_1, \mathcal{O}_2\}_{0,\lambda} + \{\mathcal{I}, \mathcal{I}, \mathcal{O}_1, \mathcal{O}_2\}_{0,0,\lambda} + \cdots$$

$$\mathcal{O}_1 \cdot \mathcal{O}_2 = \mathcal{O}_1\mathcal{O}_2 + \{\mathcal{I}; \mathcal{O}_1, \mathcal{O}_2\}_0 + \{\mathcal{I}, \mathcal{I}; \mathcal{O}_1, \mathcal{O}_2\}_{0,0} + \cdots$$

- Compute holomorphic factorization algebra for free theory
    - Feynman diagram expansion
    - Analogy: Kontsevich calculations in 2d B-model

Feynman diagrams

- Superfields $\phi_a = \phi_a^{(0)} + \phi_a^{(1)} + \phi_a^{(2)}$
    - Forms = functions on superspace
- Free action: $\frac{1}{2}\int \eta^{ab}\phi_a\bar{\partial}\phi_b \, d^2z = \int \eta^{ab}\phi_a^{(0)}\bar{\partial}\phi_b^{(1)} \, d^2z$
    - Super-propagator P is a 1-form
    - Laman graphs: 2 vertices - edges =3, greater for induced subgraphs
    - 

Universal Feynman integral

$$I_\Gamma(z_e, \lambda_v) = \int_{\mathbb{R}^{4V-4}} \bar{\partial} \left[\prod_{e=1}^{E} P_\epsilon(x_{e(0)} - x_{e(1)} + z_e, \bar{x}_{e(0)} - \bar{x}_{e(1)}) \right] \left[\prod_{v=1}^{V-1} e^{\lambda_v \cdot x_v} d^2 x_v \right]$$

- Manipulate to Fourier transform of a region in $\mathbb{R}^{2L}$
  - Manifestly UV and IR finite
- Satisfies a quadratic identity (purely geometric)
  - Implies associativity of Holomorphic Factorization Algebra
  - Bootstrappable from 1-loop

Pure gauge theory I

- Holomorphic BF theory $\int \mathrm{Tr}\, b\,(\bar{\partial} c - [c,c]) + \int \tau\, \mathrm{Tr} \partial_\alpha c\, \partial^\alpha c$
  - Coupling only affects instantons, unless position-dependent
- Free cohomology: $\mathbb{C}[b, \partial_\alpha b, \partial_\alpha \partial_\beta b, \cdots, \partial_\alpha c, \partial_\alpha \partial_\beta c, \cdots]^G$
- Tree: $Q_0 b = [c,b]$ $\qquad Q_0 c = \frac{1}{2}[c,c]$
- 1-loop: acts on 2 letters, adds 2 derivatives

Pure gauge theory II

$$S_\alpha = \mathrm{Tr}\, b\partial_\alpha c \qquad\qquad Q\, S_\alpha = \partial_\alpha(\partial_\beta c\, \partial^\beta c)$$

- Holomorphic confinement: $\partial^\alpha S_\alpha = Q\,\mathrm{Tr}\, b^2$
  - Holomorphic BF theory is secretly topological!
  - Must remain true in IR: constraint on IR behaviour
    - Compatible with confinement
- Computing low-level cohomology numerically.

Large N pure gauge theory

- Tree level: $\mathrm{Tr}\, b^n$ $\mathrm{Tr}\, b^n \,\partial_\alpha c$ $\mathrm{Tr}\, b^n \,\partial_\alpha c\, \partial^\alpha c$
- 1-loop: only gh. nr. 2 tower, no derivatives
- Holographic dual: B-model with D4 back-reaction
- Compute brackets, compare with dual?
- Can be extended to SQCD

Some future directions

- Representation theory of symmetry algebras
  - Minimal models?
- SQCD, Seiberg duality (requires non-perturbative corrections)
- Large N: compute brackets in twisted SUGRA (5d B-model)
- 3d N=2 hol-top twist, 6d holomorphic twist, etc.