The Penrose Inequality as a Swampland Condition?

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A Puzzle

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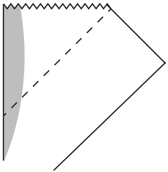
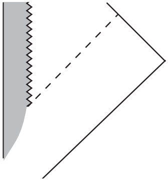
2. Fun Fact: We have never observed the formation of a singularity.
3. Fun Fact: the observable universe looks like a generic solution to the Einstein equation.

Idea for a Resolution Penrose

Nature abhors a naked singularity

In our universe, empirical evidence suggests that singularities are hidden behind event horizons. Possibly, this is a general fact about gravity: that the endpoint of matter collapse is always a black hole, never a naked singularity.





Formalizing Cosmic Censorship

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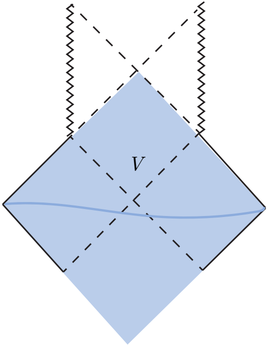
Singularities can often be diagnosed locally in terms of gravitational lensing or curvature invariants. Horizons are teleological: they require knowledge of the structure of $\mathscr{I}^+$. Not clear how to formalize the connection between the two, much less prove it.

Also overlaps with another difficult problem: proving conditions for matter to actually collapse into a singularity – Thorne's Hoop Conjecture.

Preliminary Definition:

Need the following definition first:

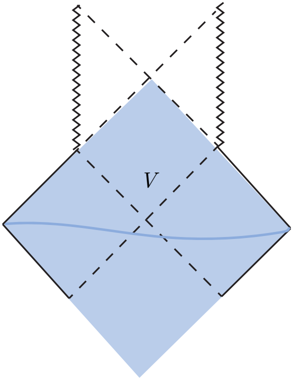
Strong asymptotic predictability (SAP): An asymptotically flat spacetime (M, g) is SAP if there exists an open region V in M such that $\overline{M \cap J^-[\mathscr{I}^+]} \subset \widetilde{V}$, and $\widetilde{V}$, the conformal completion of V, is globally hyperbolic; meaning, $\widetilde{V}$ is given by the maximal Cauchy development of some hypersurface.



Weak Cosmic Censorship Conjecture: AF

Weak Cosmic Censorship in AF space Geroch, Horowitz

The maximal Cauchy development of a set of regular, generic, asymptotically flat initial data (Σ, h, K) satisfying the Einstein constraint equations is strongly asymptotically predictable. In particular, it generates a complete (conformally inextendible) conformal boundary.

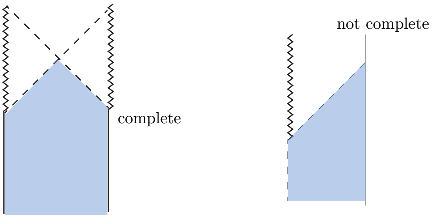


WCCC in AdS

Can generalize SAP to AAdS spacetimes by requiring that $\widetilde{V}$ be AdS hyperbolic, meaning globally hyperbolic once asymptotic boundary conditions are imposed.

Weak Cosmic Censorship in AdS Horowitz, Santos

The maximal Cauchy development of a set of regular, generic, asymptotically AdS initial data (Σ, h, K) *together with boundary conditions at* $\mathscr{I}$ is strongly asymptotically predictable. In particular, it generates a complete (conformally inextendible) conformal boundary.



Violations of CC

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- But recent counterexamples Crisford-Horowitz-Santos, others are different: the curvature grows without bound in an extended region. Perhaps CC is altogether false?

- This might be good from the perspective of observational evidence for QG, but it would be problematic for many foundational results in GR: e.g. the Hawking area theorem.

QG to the Rescue?

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Possible hypothesis?

Might suggest that even though CC **can** be badly violated in solutions to General Relativity, it **won't** be violated (except very "mildly") in solutions to General Relativity that admit a UV completion.

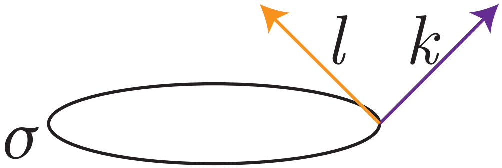
Very mildly?

What exactly do we mean by "very mild" violations of WCCC?

Intuitively speaking, we might mean that extended regions of high curvature are still hidden behind event horizons, so that the classical results we know and love (like Hawking's area theorem) are always true (in the classical regime).

One condition that is necessary for this – and possibly even sufficient – is that trapped surfaces lie behind event horizons.

Trapped Surfaces



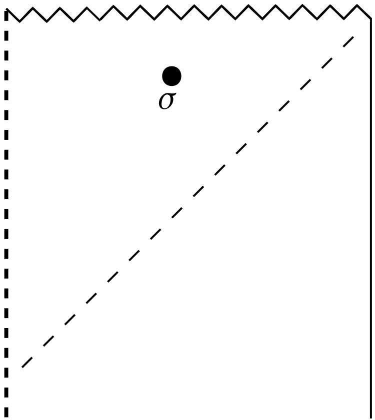
A trapped surface is defined as a compact codim-two surface satisfying:

$$\theta_k \propto \frac{d\text{Area}}{d\lambda_k} < 0$$
$$\theta_\ell \propto \frac{d\text{Area}}{d\lambda_\ell} < 0$$

Intuitively, trapped surfaces signal high spacetime curvature; concretely, the singularity theorems guarantee a singularity in the (not-too-distant) future of a trapped surface.

Trapped Surfaces Behind Horizons

If trapped surfaces were to *always* lie behind horizons, any singularities predicted by the singularity theorems would also lie behind horizons:



SAP (and the null energy condition) implies that trapped surfaces always lie behind horizons.

Possible Reformulation?

Trapped Surfaces for CC

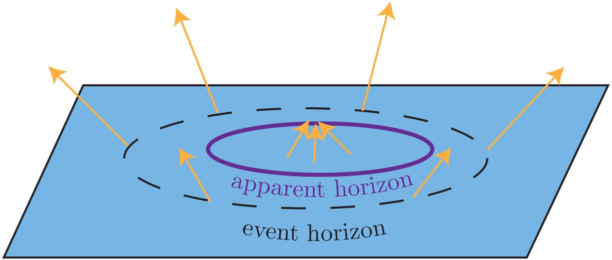
One proposal that captures the idea that signals of strong gravity should be hidden behind event horizons but that mild phenomena like Gregory-Laflamme should happen is that trapped surfaces always lie behind event horizons in the classical limit.

Another way to say this is that the outermost trapped surface lies behind an event horizon.

Apparent Horizons Behind Event Horizons

Apparent Horizon

An apparent horizon is the outermost surface on a Cauchy slice such that $\theta_k = 0$ and $\theta_\ell < 0$.



For stationary BHs, the event horizon can be foliated into surfaces with $\theta_k = 0$ and $\theta_\ell < 0$.

The Penrose Inequality: A Litmus Test for WCCC

Penrose Inequality

Given an apparent horizon σ and an AF (or AAdS) spacetime with ADM mass M:

$$\text{Area}[\sigma] \leq \text{Area}[\text{Static (AdS) BH w/ mass } M]$$

Can be proved by assuming:

1. Cosmic censorship
2. Black holes eventually settle down to Kerr-Neumann(-AdS)
3. Technical assumptions about the surface σ (i.e. a slight refinement of an apparent horizon)

Current Status of the Penrose Inequality

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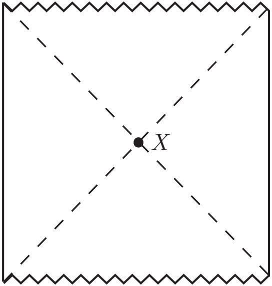
- But maybe for AAdS spacetimes with a UV completion?

Approach: Use AdS/CFT

Really, only one dictionary entry: holographic entanglement entropy (at leading order in $1/N$):

$$S_{vN}[\rho_R] = \frac{\text{Area}[X_R]}{4G\hbar}$$

where X_R is the minimal area surface such that (1) there exists a hypersurface H_R whose boundary in the conformal completion is the union $X_R \cup R$ (2) is a stationary point of the area functional.
E.g. when $R = \mathscr{I}$:



The Penrose Inequality in AdS/CFT

Holography Implies an AdS Penrose Inequality NE, Horowitz

Let σ be an apparent horizon in an asympt. AdS spacetime $(\mathcal{M}, g)$ with mass M. Assuming $(\mathcal{M}, g)$ satisfies the holographic entanglement entropy proposal:

$$\text{Area}[\sigma] \leq \text{Area}[\text{Static AdS BH with mass } M].$$

Penrose Inequality as a Swampland Condition

A Constraint on Semiclassical Solutions

If there are indeed good classical initial data sets that violate the Penrose Inequality, they cannot admit a UV completion within string theory insofar as AdS/CFT is realized within string theory.

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Question: what about the more general conjecture that trapped surfaces must lie behind event horizons?

Constraints on Trapped Surfaces from AdS/CFT

NE, Folkestad

Trapped Surfaces Lie Behind Event Horizons in Holographic spacetimes

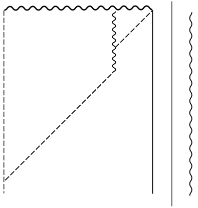
If there exists a trapped surface* τ in a classical asymptotically AdS spacetime (M, g) satisfying the NEC, then at least one of the following holds:

1. (M, g) has an event horizon, and τ lies behind it;
2. Classical GR admits solutions with 'evaporating singularities';
3. (M, g) has no holographic dual.

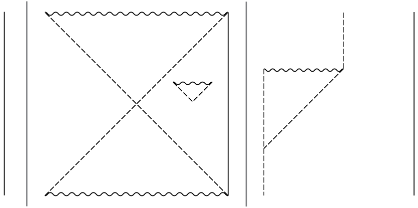
* = a certain generic condition that τ lies behind a trapped surface which is homologous to $\mathcal{I}$ and where the outgoing component of $\partial J^{-}[\tau]$ is well-behaved; no requirements on the future of τ.

“Evaporating Singularities”

Our assumptions are mostly mild, save for one: we assume that evaporating singularities do not form in classical GR. That is, any naked singularity does not suddently “end”:



Non-evaporating



Evaporating

Evaporating Singularities: Technical definition

Evaporating Singularities

An asymptotically AdS spacetime (M, g) is said to be devoid of evaporating singularities if t. For every closed set K in M, if ∂K is compact in the conformal completion, then K is compact in the conformal completion.

(This rules out inextendible curves being imprisoned in K: so following an inextendible geodesic in K means it either leaves through ∂K or goes to the asymptotic boundary. i.e. it rules out geodesic incompleteness between complete hypersurfaces that do not touch singularities.)

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- Seems like good evidence for a version of cosmic censorship in QG; but what version?
- Is the Penrose inequality, which can be checked just from initial data, a genuinely new condition on classical limits of QG?
- Work in progress by Folkestad: preliminary indications that the Penrose Inequality may be violated by AAdS spacetimes satisfying *both* the NEC and the WGC.