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Geometry of Higher-Form Structures in String Theory

Mirjam Cvetič

Penn
University of Pennsylvania

CAMTP
University of Maribor

CERN

Univerza v *Ljubljani*
Fakulteta za *matematiko in fiziko*

Motivation

Studying M/string theory on special holonomy spaces X:

- Non-compact spaces X →
  Geometric engineering of supersymmetric quantum field theories (SQFTs):

  Build dictionary:

  {operators, symmetries} ⟺ {geometry, topology}

  Focus: higher-form global symmetries
  (associated with ``flavor'' branes) ⟺ topology

- Compact spaces X →
  Quantum field theory (QFT) w/ gravity →
  Higher-form symmetries gauged or broken
  [Physical consistency conditions: swampland program]

Higher-form symmetries in (S)QFT - active field of research

[Gaiotto, Kapustin, Seiberg, Willet, 2014],...

c.f., talks by Lakshya Bhardwaj, Dan Freed

Higher-form symmetries & geometric engineering

[Del Zotto, Heckman, Park, Rudelius, 2015],...

[Morrison, Schäfer-Nameki, Willett, 2020],
[Albertini, Del Zotto, Garcia Etxebarria, Hosseini, 2020],...

[Del Zotto, Ohmori, 2020],[Apruzzi, Dierigl, Lin, 2020],
[M.C., Dierigl, Lin, Zhang, 2020],..
[Apruzzi,Bhardwaj, Oh, Schäfer-Nameki, 2021],
[Apruzzi, Bhardwaj, Gould, Schäfer-Namek, 2021],
[Bhardwaj, Giacomelli, Hübner, Schäfer-Nameki, 2021],
[M.C., Dierigl, Lin, Zhang, 2021],...
[Del Zotto,Heckman, Meynet, Moscrop, Zhang, 2022],

[M.C., Heckman, Hübner, Torres, 2022],
[Del Zotto, Garcia Etxebarria, Schäfer-Nameki, 2022],

[Hübner, Morrison, Schäfer-Nameki,Wang, 2022]
[Heckman, Lawrie, Lin, Zhang, Zoccarato, 2022],...

c.f., also talks by Muyang Liu, Paul Oehlmann in parallel session

Goals

- Identify geometric origin of higher-form symmetries for M-theory non-compact special holonomy spaces X

  Punchline: 0-form, 1-form and 2-group symmetries via cutting and gluing of singular boundary geometries

- Examples: M-theory on non-compact elliptically fibered Calabi-Yau n-folds [building on toric orbifold results]

  A few statements about compact examples [dual to F-theory compactification]

Based on

- M. C., J. J. Heckman, M. Hübner and E. Torres, ``0-Form, 1-Form and 2-Group Symmetries via Cutting and Gluing of Orbifolds," 2203.10102 non-compact examples

& work to appear compact examples

Related: Gauge symmetry topology constraints in D=8 N=1SG

- M.C., M.Dierigl, L.Lin and H.Y.Zhang, ``String Universality and Non-Simply-Connected Gauge Groups in 8d," PRL, arXiv:2008.10605 [hep-th];
- ``Higher-form Symmetries and Their Anomalies in M-/F-theory Duality," PRD, arXiv:2106.07654 [hep-th] - 8D/7D & 6D/5D
- ``Gauge group topology of 8D Chaudhuri-Hockney-Lykken vacua," PRD, arXiv:2107.04031 [hep-th];
- ``One Loop to Rule Them All: Eight and Nine Dimensional String Vacua from Junctions," arXiv:2203.03644 [hep-th]– String junctions

Outline:

- Introduction
- Defect group and higher-form symmetries → Topology of flavor symmetry group
- 2-group symmetries
- Compact (elliptic) examples → fate of higher-form symmetries
- Concluding remarks

I. Introduction

Defect group for M-theory on non-compact X

- Defect Group for extended *p*-dim operators associated with M2 and M5 branes: $\mathcal{D}_p = \mathcal{D}_p^{\mathrm{M2}} \oplus \mathcal{D}_p^{\mathrm{M5}}$

- M2, M5 in X live on relative cycles:

[Morrison, Schäfer-Nameki, Willett, 2020],
[Albertini, Del Zotto, Garcia Etxebarria, Hosseini, 2020],
[Del Zotto, Heckman, Park, Rudelius, 2015]

$$\mathcal{D}_p^{\mathrm{M2}} = \frac{H_{3-p}(X, \partial X)}{H_{3-p}(X)} \cong H_{3-p-1}(\partial X)|_{\mathrm{triv}}$$ [p-dim el. operators in SCFT]

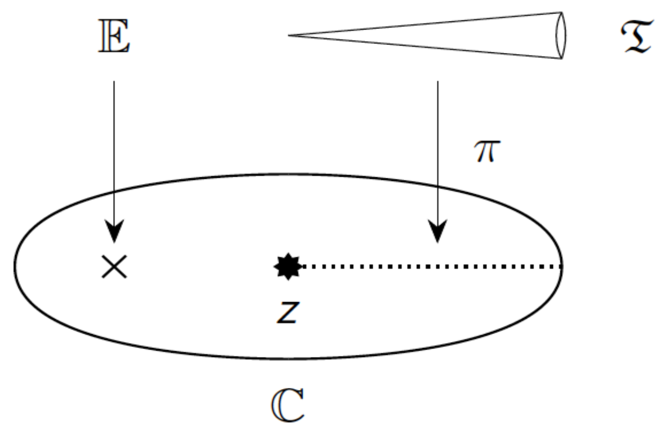
$$\mathcal{D}_p^{\mathrm{M5}} = \frac{H_{6-p}(X, \partial X)}{H_{6-p}(X)} \cong H_{6-p-1}(\partial X)|_{\mathrm{triv}}$$ [p-dim mag. operators in SCFT]

- Defect pairing $\langle \cdot , \cdot \rangle$ in s.t. ⟺ Linking Pairing $\ell(\cdot , \cdot)$ in ∂X

Example: non-compact *K3*

[M.C., Dierigl, Lin, Zhang, 2021, 2022]

- Local elliptically fibered *K3* $\mathbb{E} \hookrightarrow X \to \mathbb{C}$
- Singular fiber of Kodaira type ϕ at $z \in \mathbb{C}$ w/ monodromy M



- Boundary ∂X→ S^1
- Exact sequence for spaces fibered over circles:

$$0 \to \text{coker}\,(M_n - 1) \to H_n(X) \to \ker\,(M_{n-1} - 1) \to 0$$

M_n - monodromy in homology in degree n

$$\mathcal{D}_1^{\text{M2}} = \mathcal{D}_4^{\text{M5}} = H_2(X, \partial X)/H_2(X) \cong \text{Tor Coker}(M-1) = \langle \mathfrak{T} \rangle$$

- X engineers 7D SYM with gauge algebra g_ϕ w/
  Defect group $\mathcal{D} = \langle \mathfrak{T} \rangle_1^{\text{M2}} \oplus \langle \mathfrak{T} \rangle_4^{\text{M5}}$

Example (continued):

- Non-trivial self-linking/intersection: $\ell(\partial\mathfrak{T},\partial\mathfrak{T}) = \mathfrak{T}\cdot\mathfrak{T} \neq 0$

- Elements of $\mathcal{D}_1^{M2}$, $\mathcal{D}_4^{M5}$ typically mutually non-local

- Choose electric polarization $\mathcal{D}_1^{M2}$ [for the rest of the talk]

- Gauge group is simply connected G_ϕ w/algebra g_ϕ (ADE)

- Resulting 7D SYM theory w/ gauge group G_ϕ

- (Wilson) Line operators $\mathcal{D}_1^{M2}$ acted on by 1-form symmetry $Z_{G\phi}$ [Pontyagin dual to line operators] fixes group topology

Now, turn to higher-form structures for non-compact spaces X in higher dimensions (D ≥ 6)

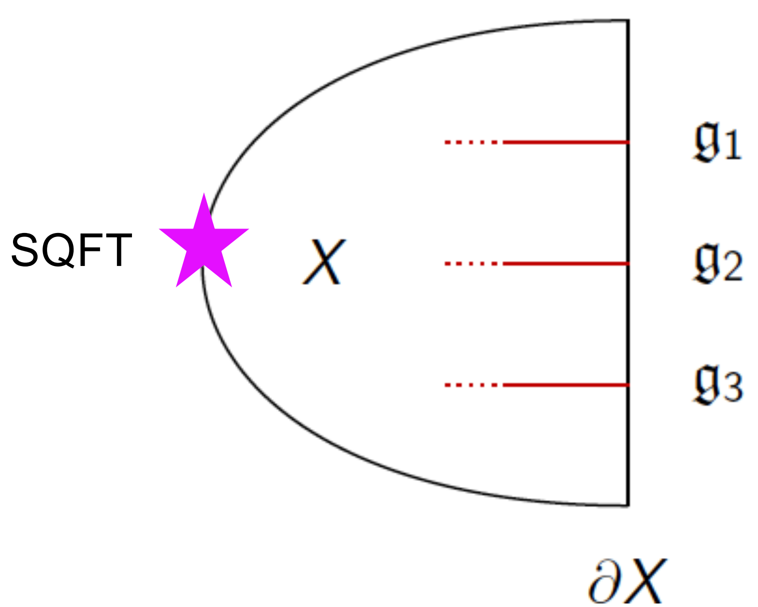
→ leads to new phenomena

II. Geometrizing Topology of Flavor Group

Non-compact ADE loci ≡ flavor branes → flavor symmetries

Naïve flavor symmetry $\widetilde{G}_F$ (simply connected w/ Lie Algebra $\mathfrak{g}_i$)

[From now on only ADE's in ∂X]

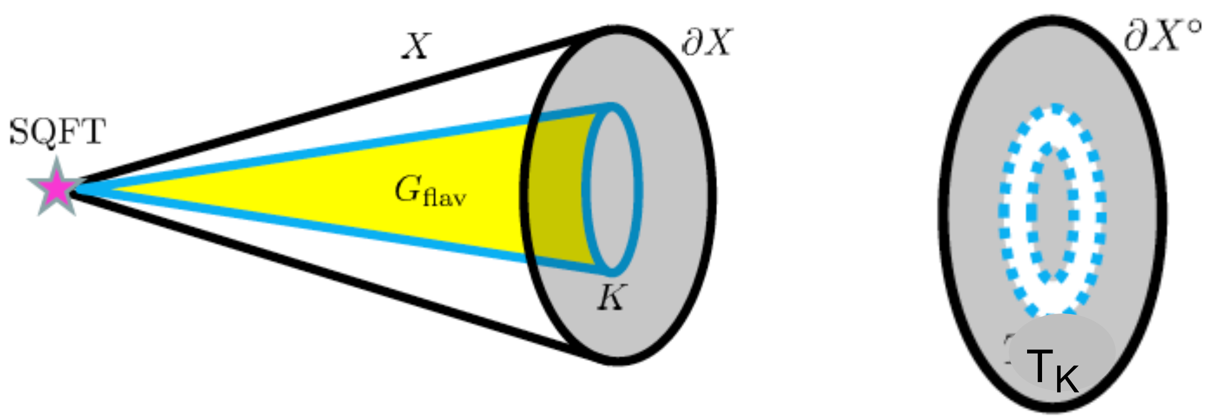


$$\widetilde{G}_F = \widetilde{G}_1 \times \widetilde{G}_2 \times \widetilde{G}_3 \times \ldots$$

(Flavor Wilson) lines → fix topology of favor symmetry G_F
from singular boundary topology

Boundary geometry of flavor branes:

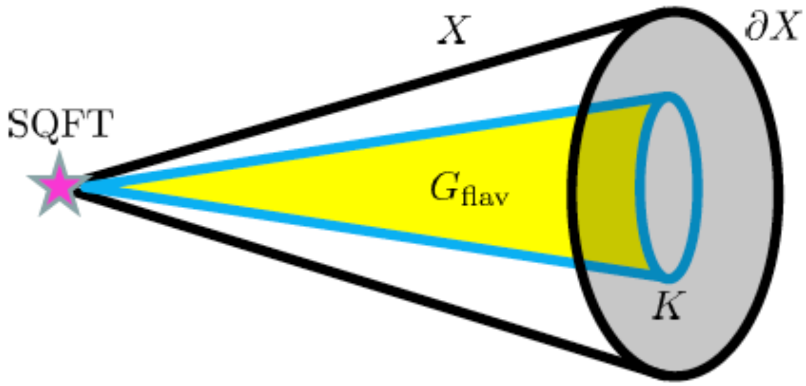
- Singular non-compact space X w/
  $K = \cup_i K_i$ - ADE loci (of flavor branes) in the boundary **∂X**
- Define a smooth boundary $\partial X^\circ = \partial X \setminus K$ &
  a tubular region T_K (excise K)
- Locally $T_K \cap \partial X^\circ \cong \cup_i K_i \times S^3/\Gamma_i$

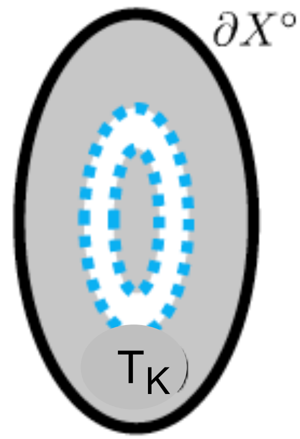


- Naïve flavor center symmetry:

$$Z_{\tilde{G}_F} = \mathrm{Tor}\, H_1(T_K \cap \partial X^\circ) \cong Z_{\tilde{G}_1} \oplus Z_{\tilde{G}_2} \oplus Z_{\tilde{G}_3} \oplus \dots$$

Boundary geometry of true flavor center symmetry Z_{G_F}





[Mayer, 1929], [Vietoris, 1930]

- Key: Mayer-Vietoris sequence in homology for singular boundary $\partial X = \partial X^\circ \cup T_K$ → obtain:

$$0 \to \underbrace{\ker(\iota_1)}_{\text{flavor center}} \to \underbrace{H_1(\partial X^\circ \cap T_K)}_{\text{naive flavor center}} \xrightarrow{\iota_1} \underbrace{H_1(\partial X^\circ)}_{\text{(un)twisted sector}} \oplus H_1(T_K) \to \underbrace{H_1(\partial X)}_{\text{1-form symmetry}} \to 0$$

naïve 1-form symmetry

$$Z_{G_F} = \mathrm{Ker}\left(\iota_1 : Z_{\widetilde{G}_F} \cong H_1(\partial X^\circ \cap T_K) \to H_1(\partial X^\circ) \oplus H_1(T_K)\right)$$

- Motivated by orbifold homology: $H_1(\partial X^\circ) = H_1^{\mathrm{orb}}(\partial X)$

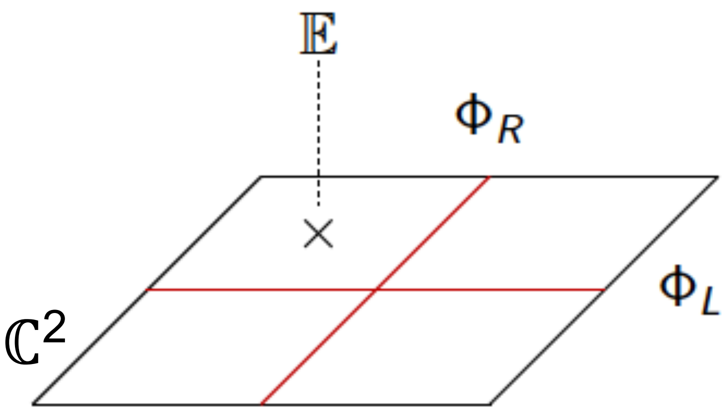
[Thurston, 1980], [Moerdijk, Pronk, 1997]

Example: Elliptically fibered threefold

$$\mathbb{E} \hookrightarrow X_3 \to B = \mathbb{C}^2$$ (non-compact base)

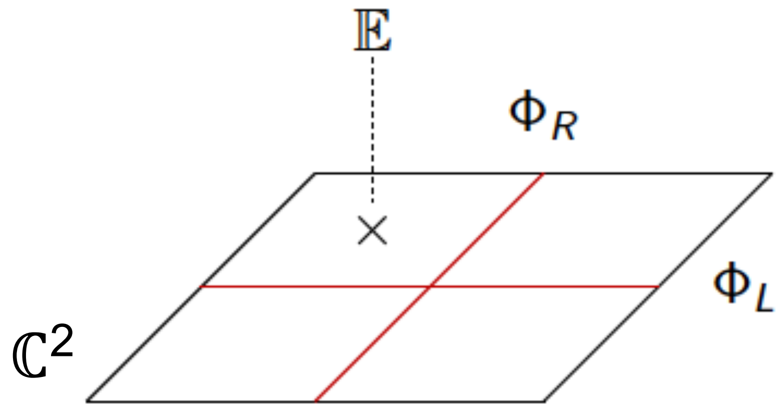
[(G_L; G_R)-Conformal Matter]
[Del Zotto, Heckman, Tomasiello, Vafa, 2014]

- Non-compact discriminant loci: ϕ_L on $\mathbb{C} \times \{0\}$ & ϕ_R on $\{0\} \times \mathbb{C}$



- Boundary five-manifold $\mathbb{E} \hookrightarrow \partial X_3 \to S^3$
- Discriminant locus consists of two linking circles S^1_L & S^1_R
- Excise singular fibers $\mathbb{E} \hookrightarrow \partial X^\circ \to S^1_L \times S^1_R$

Example (continued):



[(G_L; G_R)-Conformal Matter]

- Again, exact sequence for spaces fibered over circles $S^1{}_{L\ R}$

$$0 \rightarrow \mathrm{coker}\,(M_n - 1) \rightarrow H_n(X) \rightarrow \ker\,(M_{n-1} - 1) \rightarrow 0$$

  M_n - monodromy in homology in degree n.

- From monodromies M_L & M_R about $S^1{}_L$ & $S^1{}_R$.:

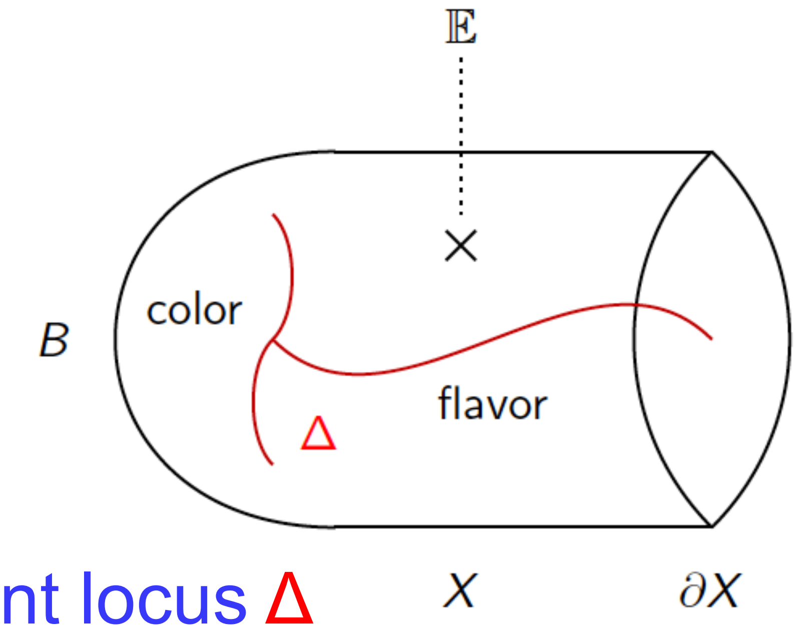
$$\mathrm{Tor}\, H_1(\partial X^\circ) = \mathrm{Tor}\, \frac{\mathbb{Z}^2}{\mathrm{Im}(M_{\Phi_L} - 1) \cup \mathrm{Im}(M_{\Phi_R} - 1)}$$

- (SU(n); SU(m)) Conformal Matter $G_F = \dfrac{SU(n) \times SU(m)}{\mathbb{Z}_{\mathrm{gcd}(n,m)}}$

- General: $G_F = \dfrac{G_L \times G_R}{Z_{\mathrm{diag}}}$

General Elliptically fibered Calabi-Yau n-folds

- $\mathbb{E} \hookrightarrow X_n \to B_{n-1}$



- Focus on non-compact discriminant locus Δ

[Hübner, Morrison, Schäfer-Nameki, Wang 2022]

- Homology groups of ∂X, ∂X° & ∂X_F = ∂X \ {singular fibers}

- Deformation retractions of ∂B \ Δ lift to ∂X_F

- Glue singular fibers back in

$$\text{Tor } H_1(\partial X^\circ) = \text{Tor } H_1(\partial X_F) \oplus \text{Tor } H_1(\partial B)$$

→ Further explicit elliptic examples; also, orbifolds, G_2,… No time

III. 2-Groups

[Benini, Cordova, Hsin, 2019],
[Lee, Ohmori, Tachikawa, 2021], . . .

- Two key short exact sequence

$$0 \to \mathcal{C} \to Z_{\widetilde{G}_F} \to Z_{G_F} \to 0$$
$$0 \to \mathcal{C}^\vee \to \widetilde{\mathcal{A}}^\vee \to \mathcal{A}^\vee \to 0$$

[& Postnikov class]

- Z_{G_F} : Center flavor symmetry
- $Z_{\widetilde{G}_F}$: Naïve center flavor symmetry
- $\mathcal{A}^\vee$: Line operators modulo screening by local operators
- $\widetilde{\mathcal{A}}^\vee$: (Naïve) Line operators modulo screening by local operators transforming in reps. of Z_{GF}
- $\mathcal{C}^\vee$: Line operators in the kernel of $\widetilde{\mathcal{A}}^\vee \to \mathcal{A}^\vee$

Pontryagin duals: $\mathcal{D}^\vee = \mathrm{Hom}(\mathcal{D}, U(1))$; [Line operators ←→ One-forms]

- Taking the Pontryagin dual (one-form symmetries) of line- operators in the second sequence & reversing arrows:

$Z_{\text{0-form}}$: $0 \to \mathcal{C} \to Z_{\widetilde{G}} \to Z_G \to 0$

1-form: $0 \to \mathcal{A} \to \widetilde{\mathcal{A}} \to \mathcal{C} \to 0$

w/ Postnikov class $P \in H^3(BG, \mathcal{A})$

[obtained via first extension $w_2 \in H^2(BZ_{\widetilde{G}}, \mathcal{C})$ &

Bockstein homomorphis $\beta : H^2(BZ_G, \mathcal{C}) \to H^3(BZ_G, \mathcal{A})$]

- Sufficient: when second sequence does not split, i.e., $\tilde{\mathcal{A}} \neq \mathcal{A} \oplus C$ → 2-group

⇓

- One can collapse the two sequences:

2-group: $0 \to \mathcal{A} \to \widetilde{\mathcal{A}} \to Z_{\widetilde{G}_F} \to Z_{G_F} \to 0$

2-groups and Mayer-Vietoris

- Derive parallel homology sequences (physically motivated):

$$0 \to \ker(\iota_1) \to H_1(\partial X^\circ \cap T_K) \xrightarrow{\iota_1} \frac{H_1(\partial X^\circ \cap T_K)}{\ker(\iota_1)} \to 0\,,$$

$$0 \to \frac{H_1(\partial X^\circ \cap T_K)}{\ker(\iota_1)} \to H_1(\partial X^\circ) \oplus H_1(T_K) \to H_1(\partial X) \to 0$$

- Precisely the 2-group sequences!

- Again, when the bottom sequence does not split → 2-group mixing 0-form and 1-form (flavor) symmetries

IV. Compact Models

- Compact singular space X → theory that includes quantum gravity & global symmetries gauged or broken
- What is M-theory gauge group?
- Elliptically fibered geometries (via M/F-theory duality):

- Non-Abelian group algebras – ADE Kodaira classification
Group topology → Mordell-Weil torsion
[Aspinwall, Morrison, 1998], [Mayrhofer, Morrison, Till, Weigand, 2014], [M.C., Lin, 2017]

- Abelian groups → Mordell-Weil ``free'' part
[Morrison, Park 2012], [M.C., Klevers, Piragua, 2013], [Borchmann, Mayrhofer, Palti, Weigand, 2013]...

- Total group topology → Shoida map of Mordell-Weil
[M.C., Lin, 2017]

$$\frac{U(1)^r \times G_{\text{non-ab}}}{\prod_{i=1}^{r} \mathbb{Z}_{m_i} \times \prod_{j=1}^{t} \mathbb{Z}_{k_j}}$$

Digression: F-theory on elliptically fibered Calabi-Yau fourfolds w/specific elliptic fibration (F_{11} polytope) led to D=4 N=1effective theory

[M.C., Klevers, Peña, Oehlmann, Reuter, '15]

Standard Model gauge group $$\frac{SU(3) \times SU(2) \times U(1)}{\mathbb{Z}_6}$$

w/ gauge group topology
(geometric - encoded in Shioda Map of MW)

[M.C., Lin, '17]

⬇ w/ toric bases B_3 (3D polytopes)

Quadrillion Standard Models (QSMs)
with 3-chiral families & gauge coupling unification

[gauge divisors – in class of *anti-canonical divisor K_C*]

[Bies, M.C., Donagi,(Liu), Ong, '21,'22]

Current efforts: determination the exact matter spectra
(including vector pair & # of Higgs pairs)
→ studies of root bundles on matter curves

c.f., Martin Bies' talk in parallel session (slides posted)

How to relate these results, encoded in arithmetic structure of elliptic curve – Mordell Weil, to higher-form symmetries?

- Global symmetries, including higher-form ones should be gauged or broken [No Global Symmetry Hypothesis]

  ...[Harlow, Ooguri '18]

- In 8D N =1 Supergravity:
  quantified conditions under which no anomalies due to gauged 1-form symmetry [magnetic version]

  [M.C., Diriegl, Ling, Zhang, '20]

- True for all 8D N=1 string compactifications (beyond F-theory) via (refined) string junction construction

  [M.C., Diriegl, Ling, Zhang, '22]

Fate of higher-form structures in Compact Geometries:

- Decompose $X = \cup_n X_n$ into local models X_n

  Converse:

  Glue $\{X_n\}$ to X ←→ Couple $\{SQFT_n\}$ to resulting one

  & includes gravity

- Relative Cycles in X_n compactify →

  (some) defects in $SQFT_n$ become dynamical - ``gauged”

Fate of higher-form structures in Compact Geometries (continued):

- Mayer-Vietoris Sequence for covering $\{X_n\}$:

$$\partial_2 : H_2(X) \rightarrow \bigoplus_n H_1(\partial X_n)$$

- Decomposition of compact two-cycles into a sum of relative cycles associated with each local model

- Elliptically fibered geometries:
  torsional cycles associated w/ Mordell-Weil group
  decomposition into relative cycles of $\{X_n\}$

- Arguments extend beyond elliptically fibered models:
  all T^4/Γ_i , some T^6/Γ_i ,...

[M. C., Heckman, Hübner, Torres, to appear]

Thank you!