Giant Gravitons in Twisted Holography

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Based on arxiv:2106.14859, arxiv:2207.xxxxx with Davide Gaiotto

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Twisting Supersymmetric QFTs

Twisting SQFT: restricting to a cohomology of a supercharge

Physics motivation: produces a protected subsector of SQFT that is easier to study

Examples:

- A and B topological twists of 2d $\mathcal{N} = 2$ sigma models [Witten '91]
- holomorphic twist of 4d $\mathcal{N} = 1$ [Johansen '94] [Nekrasov '96] [Costello '13] [Gwilliam, Williams '18] [Saberi, Williams '19] [Davide Gaiotto's talk]
- 2d chiral algebra subsector of 4d $\mathcal{N} = 2$ SCFT [Beem, Lemos, Liendo, Peelaers, Rastelli, Rees '13]

Twisted holography: holographic duals of these twists

Twisted Holography

Example: protected subsector of AdS$_5$/CFT$_4$ [Costello, Gaiotto '18]:

$\mathcal{N} = 4$ SYM with $U(N)$ $\approx$ type IIB on AdS$_5 \times S^5$

[Beem et al.] twist $\downarrow$

$\downarrow$ twisted strings/SUGRA [Costello, Li]

chiral algebra $\mathcal{A}_N$ $\approx$ B-model on $SL(2,\mathbb{C}) \approx$ AdS$_3 \times S^3$

Motivation:

- Simplifications
- Connections to math

Twisted Holography

Examples of twisted supergravity/superstrings:

- Twisted type IIA and IIB supergravity [Costello, Li '16] [Saberi, Williams '21]
- Twisted 11d supergravity [Saberi, Williams '21] [Eager, Hahner '21] [Raghavendran, Saberi, Williams '21]
- Twisted S-duality [Raghavendran, Yoo '19]

Other examples of twisted holography:

- D2-D4 system in 6d topological strings [Ishtiaque, Moosavian, Zhou '18]
- Subsector of duality between SUGRA on AdS$_3 \times S^3 \times T^4$ and symmetric orbifold $Sym^N(T^4)$ [Costello, Paquette '20]
- M2 and M5 branes in an Ω-background [Costello '16 '17]

In this talk

- Review the duality [Costello, Gaiotto '18]

  2d chiral algebra $\mathcal{A}_N \approx$ topological B-model on $SL(2,\mathbb{C})$

- Match saddles of determinant correlation functions with D1-brane configurations

  - Spectral curve construction

- Extend the match to non-conformal vacua of the chiral algebra

- Future directions: "bootstraping" to AdS$_5 \times S^5$, holomorphic twist

Twisted Holography

Twisted holography example [Costello, Gaiotto '18]:

B-model topological strings on deformed conifold $SL(2,\mathbb{C})$ with coupling N^{-1}	$\approx$	large N expansion of chiral algebra $\mathcal{A}_N =$ gauged $\beta\gamma$ system in adj. of $U(N)$

Simplifications:

- (almost) free field theory computations in the chiral algebra $\mathcal{A}_N$
- D1-branes are **holomorphic curves** in $SL(2,\mathbb{C})$
- dependence on t'Hooft coupling drops out

Chiral algebra $\mathcal{A}_N$

The chiral algebra subsector of $\mathcal{N} = 4$ SYM is a **gauged $\beta\gamma$ system** [Beem, Lemos, Liendo, Peelaers, Rastelli, Rees '13]:

Symplectic bosons X, Y in the adjoint of $U(N)$:

$$X^a_b(z) Y^c_d(w) \sim \delta^a_d \delta^c_b \frac{1}{N} \frac{1}{z - w}$$

For the future, define a linear combination:

$$Z(u; z) \equiv X(z) + uY(z)$$

BRST charge:

$$Q_{\text{BRST}} \sim N \oint \text{Tr}\left(c[X, Y] + \frac{1}{2} b[c, c] \right)$$

B-model/Kodaira-Spencer theory

- The chiral algebra $\mathcal{A}_N$ is dual to **topological B-model on $SL(2,\mathbb{C})$**

- The spacetime theory is **Kodaira-Spencer (BCOV) theory**
  [Bershadsky, Cecotti, Ooguri, Vafa '93 '94]
  - holomorphic, depends only on complex structure (no metric)

- The chiral algebra $\mathcal{A}_N$ is supported by N B-branes wrapping $\mathbb{C} \subset \mathbb{C}^3$

"Derivation" [Costello, Gaiotto '18]:

$$\text{B-model on } \mathbb{C}^3 + N \text{ D1-branes} \longrightarrow \text{B-model on } SL(2,\mathbb{C}) \approx \text{AdS}_3 \times S^3$$

$$\uparrow$$

$$\mathcal{A}_N$$

Holographic dictionary

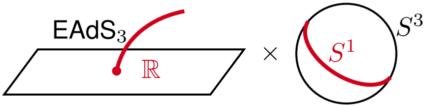
Chiral algebra $\mathcal{A}_N$	B-model on $SL(2,\mathbb{C})$
single traces	local modifications of asymptotic boundary condition
(sub)determinants	Giant Graviton D1-branes

In particular, determinant operator

$$\det(m + Z(u;z)), \qquad Z(u;z) = X(z) + uY(z)$$

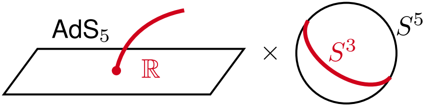
is dual to

- z = position at the boundary of AdS_3
- u = orientation of $S^1 \subset S^3$
- m controls size of $S^1 \subset S^3$



Giant Gravitons

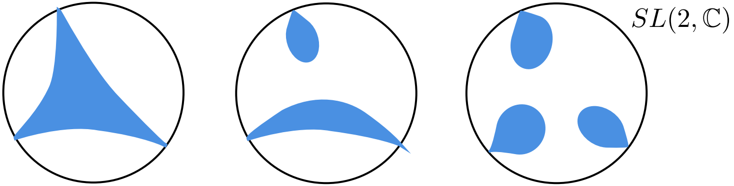
Determinant operators in $\mathcal{N} = 4$ SYM are dual to Giant Graviton D3-branes in $\text{AdS}_5 \times S^5$:



Future direction: ”bootstrap” D3-branes from B-model D1-branes

Determinants and Giant Gravitons

Many possible brane configurations with the same boundary behaviour



We will match saddles ρ^* of correlation functions of determinants with brane configurations

- m_i, u_i, z_i control boundary behaviour
- saddles ρ^* will control the shape in the bulk

Determinant correlation functions

[Jiang, Komatsu, Vescovi '19]

see also [Chen, de Mello Koch, Kim, Van Zyl '19] [Berenstein, Wang '22]

- Fermionize determinants

$$\mathcal{D}(m;u;z) \equiv \det(m + Z(u;z)) = \int [\mathrm{d}\bar{\psi}\mathrm{d}\psi]\, e^{\bar{\psi}(m+Z(u,z))\psi}$$

- Rewrite correlation function using auxiliary bosonic variables ρ^i_j for $i \neq j$, $\rho^i_i \equiv m_i$

$$\Big\langle \prod_i \mathcal{D}(m_i;u_i;z_i) \Big\rangle \sim \int [\mathrm{d}\rho]\, e^{\frac{N}{2}\sum_{i\neq j} \frac{z_i - z_j}{u_i - u_j}\rho^i_j \rho^j_i}\ (\det \rho)^N$$

- Saddle point analysis

$$\Big\langle \prod_i \mathcal{D}(m_i;u_i;z_i) \Big\rangle \sim \int [\mathrm{d}\rho]\ e^{N\, S[\rho]}$$

with action

$$S[\rho] = \frac{1}{2}\sum_{i\neq j} \frac{z_i - z_j}{u_i - u_j}\rho^i_j \rho^j_i + \log\det\rho$$

Saddles and branes

Saddle point equations in the matrix form:

$$[\zeta, \rho] + [\mu, \rho^{-1}] = 0$$

where

$$\zeta = \begin{pmatrix} \mathsf{z}_1 & & \\ & \ddots & \\ & & \mathsf{z}_k \end{pmatrix}, \quad \mu = \begin{pmatrix} \mathsf{u}_1 & & \\ & \ddots & \\ & & \mathsf{u}_k \end{pmatrix}, \quad \rho = \begin{pmatrix} \mathsf{m}_1 & & ? \\ & \ddots & \\ ? & & \mathsf{m}_k \end{pmatrix}$$

We will match saddles ρ^* to classical brane configurations in B-model on $SL(2, \mathbb{C})$.

For each ρ^* we will define a **spectral curve** S_{ρ^*} in $SL(2, \mathbb{C})$ and check it matches dual Giant Graviton brane.

Spectral curve

For each saddle ρ^* we define a spectral curve S_{ρ^*}:

- Define **commuting matrices**:

$$B(a) = a\mu - \rho, \quad C(a) = a\zeta + \rho^{-1}, \quad D(a) = a\zeta\mu + \rho^{-1}\mu - \zeta\rho,$$

which satisfy

$$aD(a) - B(a)C(a) = 1.$$

- Define spectral curve:

$$\mathcal{S}_{\rho^*} = \Big\{(a, b, c, d) \quad \text{s.t. } b, c, d \text{ are } \textbf{simultaneous eigenvalues} \text{ of } B(a), C(a), D(a)\Big\}$$

- $\mathcal{S}_{\rho^*}$ comes with a natural line bundle :

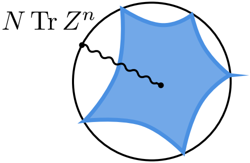
$$\mathcal{L}_{\rho^*} = \text{ common eigenline of } B(a), C(a), D(a)$$

Holographic checks

Boundary behaviour $a \to \infty$:

$$\frac{B(a)}{a} = \begin{pmatrix} u_1 - \frac{m_1}{a} & & \\ & \ddots & \\ & & u_k - \frac{m_k}{a} \end{pmatrix} + \ldots, \qquad \frac{C(a)}{a} = \begin{pmatrix} z_1 + \frac{p_1}{a} & & \\ & \ddots & \\ & & z_k + \frac{p_k}{a} \end{pmatrix} + \ldots,$$

where $p_i \equiv \partial S / \partial m_i = [\rho^{-1}]^i_i$.



Various holographic checks:

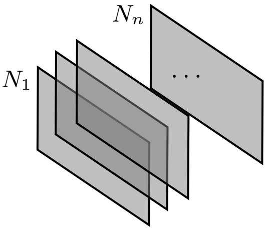
- Correlation functions of determinants with a **single-trace**
- Modifications of determinant / excitations of the brane

Coulomb branch geometries

- Duality can be extended to **non-conformal vacua** of the chiral algebra $\mathcal{A}_N$
- Twisted analog of [Kraus, Larsen, Trivedi '98]:

$$\text{Coulomb branch of } \mathcal{N}=4 \text{ SYM} \longleftrightarrow \text{multi-center solutions}$$

- Dual Calabi-Yau geometries are **deformations of $SL(2,\mathbb{C})$**
- Holographic check:
  - Determinant correlation functions (with a single-trace) and dual Giant Graviton branes



Vacua of chiral algebras

Translation-invariant vacuum $\mathcal{V}$ of a chiral algebra: vevs $\langle \mathcal{O}_a(0) \rangle_{\mathcal{V}}$

$$\langle \mathcal{O}_a(z)\mathcal{O}_b(0) \rangle_{\mathcal{V}} = \sum_n z^{-n} \langle [\mathcal{O}_a\mathcal{O}_b]_n(0) \rangle_{\mathcal{V}}$$

By cluster decomposition for $z \to \infty$:

$$\langle [\mathcal{O}_a\mathcal{O}_b]_n(0) \rangle_{\mathcal{V}} = 0, \qquad n \leq -1$$

The quotient by the above is **the C_2-algebra of the VOA** [Zhu '96]

It is a Poisson algebra with product $[O_a\mathcal{O}_b]_0$, which commutes with taking a vev:

$$\langle [\mathcal{O}_a\mathcal{O}_b]_0(0) \rangle_{\mathcal{V}} = \langle \mathcal{O}_a(0) \rangle_{\mathcal{V}} \langle \mathcal{O}_b(0) \rangle_{\mathcal{V}}$$

This gives a map:

space of vacua $\mathcal{V}$ $\longleftrightarrow$ **reduced spectrum of the C_2-algebra**

Vacua of chiral algebras

- **Physical interpretation:**

  The reduced spectrum of the Zhu's C_2-algebra
  (the associated variety of a VOA) is the space of (translation-invariant) vacua of the VOA.

- In case of **chiral algebras of $\mathcal{N} = 2$ SCFT** [Beem, Rastelli '17]:

  Higgs branch of $\mathcal{N} = 2$ SCFT $\leftrightarrow$ the associated variety of the VOA

- Higgs branch of $\mathcal{N} = 2$ SCFT maps to the moduli space of its chiral algebra subsector

Future directions

- Spectral curve construction in other examples of twisted or free field holography
- Find SUSY D3-branes in AdS$_5 \times S^5$ that correspond to our B-model D1-branes
- Consider $\mathcal{O}(N^2)$ operators eg. $(\det Z)^N$, which are dual to backreacted geometries
- Analyze which saddles actually contribute
- Holographic dual of the **holomorphic twist of $\mathcal{N} = 4$ SYM**
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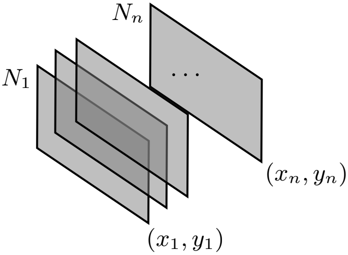
Thank you!

Coulomb branch geometries

Coordinates (x, y, z) in $\mathbb{C}^3$

N_i parallel B-branes wrapping $\mathbb{C}$'s at

$$x = x_i, \quad y = y_i \qquad i = 1, \dots n$$



Deformed geometry can be described in patches: $\forall i\ x \neq x_i$ or $y \neq y_i$

Coordinate transformation between patches I and I':

$$z_I = z_{I'} + \frac{N_i/N}{(x - x_i)(y - y_i)}$$

For standard $SL(2, \mathbb{C})$ geometry:

$$z_0 - z_\infty = \frac{1}{xy}$$