Magnetic Quivers for Symplectic Singularities

String Math 2022, University of Warsaw, Poland

Antoine Bourget

IPhT, CEA, Saclay
Ecole Normale Supérieure, Paris

July 15, 2022

Based on long time collaboration with P. Argyres, M. van Beest, S. Cabrera, A. Dancer, S. Giacomelli, J. Grimminger, A. Hanany, R. Kalveks, F. Kirwan, M. Martone, S. Schäfer-Nameki, M. Sperling, G. Zafrir, Z. Zhong...

Mathematical Motivation

Symplectic singularities [Beauville 00] appear in many contexts, in physics and mathematics:

- Kleinian singularities $\mathbb{C}^2/\Gamma$.
- HyperKähler geometry (in particular HyperKähler quotients $M /\!/\!/ G$)
- (Normalization of) Closures of nilpotent orbits of simple Lie algebras
- Transverse slices in affine Grassmannians
- Higgs branches of supersymmetric gauge theories
- 3d $\mathcal{N} = 4$ Coulomb branches

The goal of this talk is to

- Define a combinatorial tool (the *magnetic quiver*, MQ) to characterize a symplectic singularity X.
- Explain how the MQ can be used to understand the structure of X (isometries, symplectic leaves stratification, coordinate ring structure)
- Explain how one can compute the MQ in specific examples.

Physical Motivation

How to explore the landscape of SCFTs?

Higgs branches of SCFTs with 8 supercharges (in $d = 3, 4, 5, 6$) are symplectic singularities.

$$\boxed{\text{SCFT (8 susy)}} \longrightarrow \boxed{\text{Symplectic Singularity}}$$

$\rightarrow$ Explore the landscape of symplectic singularities.

Today: **magnetic quivers** as a new tool for that exploration.

Plan

1. What are Magnetic Quivers?
   - Example 1: closure of minimal nilpotent orbits
   - Example 2: 4d $\mathcal{N} = 2$ SCFTs
2. What do they tell us?
   - Coordinate ring
   - The Stratification
   - Example 3: Slices in the Affine Grassmannian
3. How to derive them?
   - Example 4: 5d SCFTs and toric geometry

Symplectic Singularities

General idea:

Smooth		Singular
Calabi-Yau Manifolds	$\rightarrow$	Gorenstein Singularities
HyperKähler Manifolds	$\rightarrow$	Symplectic Singularities

Definition. A complex algebraic variety X is a *symplectic variety* if it is normal and if its smooth part admits a holomorphic symplectic form ω such that its pull-back to any resolution $\widetilde{X} \rightarrow X$ extends to a holomorphic 2-form on $\widetilde{X}$.

[Beauville 00]

In this talk: A *conical symplectic singularity* (CSS) is an affine symplectic variety with a $\mathbb{C}^*$-action with positive weights:

$$\mathbb{C}[X] = \bigoplus_{n \geq 0} \mathbb{C}[X]_n \,, \qquad \mathbb{C}[X]_0 = \mathbb{C} \,.$$

We assume ω has weight -2.

Superconformal Algebras with 8 supercharges

Dimension	Susy	Bosonic subalgebra		SCA
$d=6$	$\mathcal{N}=(1,0)$	$\mathfrak{so}(6,2)\oplus\mathfrak{su}(2)_H$	$\subset$	$\mathfrak{osp}(6,2\vert 1)$
$d=5$	$\mathcal{N}=1$	$\mathfrak{so}(5,2)\oplus\mathfrak{su}(2)_H$	$\subset$	$\mathfrak{f}(4)$
$d=4$	$\mathcal{N}=2$	$\mathfrak{so}(4,2)\oplus\mathfrak{su}(2)_H\oplus\mathfrak{u}(1)_C$	$\subset$	$\mathfrak{su}(2,2\vert 2)$
$d=3$	$\mathcal{N}=4$	$\mathfrak{so}(3,2)\oplus\mathfrak{su}(2)_H\oplus\mathfrak{su}(2)_C$	$\subset$	$\mathfrak{osp}(4\vert 4)$

SCFTs are

- "Rare" in 6d / 5d – isolated, rely on exceptional isomorphisms, non Lagrangian.
- More common in 4d (Lagrangian, class S, Argyres-Douglas...).
- Very large number in 3d. Includes IR limits of **quiver gauge theories**.

In all cases, existence of **Moduli space of vacua**, always contains Higgs branch, which is a symplectic singularity.

Magnetic Quivers

The Coulomb branch of a 3d $\mathcal{N} = 4$ SCFT is a CSS, due to $\mathfrak{su}(2)_C$. Physically, parametrized by dressed monopole operators.

[Gaiotto, Witten 09]

[Cremonesi, Hanany, Zaffaroni 14], [Bullimore, Dimofte, Gaiotto 15]

[Nakajima 15], [Braverman, Finkelberg, Nakajima 15]

A **magnetic quiver** is a combinatorial way to encode a CSS.

$$\begin{tikzcd}
\boxed{\text{Good Quiver}} \arrow[r] \arrow[d] & \boxed{3\text{d } \mathcal{N} = 4 \text{ SCFT}} \arrow[d, "3\text{d } \mathcal{N} = 4 \text{ CB}"] \\
\boxed{\text{Generalized Quiver}} \arrow[r, "\mathcal{C}"] & \boxed{\text{CSS}} \arrow[l, bend left, dotted, "\text{Magnetic Quiver}"]
\end{tikzcd}$$

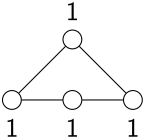
A (generalized) quiver Q is a *magnetic quiver* for the CSS X if $\mathcal{C}(Q) = X$.

Outline

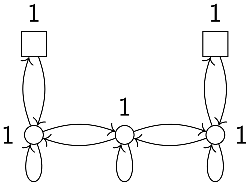
1. What are Magnetic Quivers?
   - Example 1: closure of minimal nilpotent orbits
   - Example 2: 4d $\mathcal{N} = 2$ SCFTs
2. What do they tell us?
   - Coordinate ring
   - The Stratification
   - Example 3: Slices in the Affine Grassmannian
3. How to derive them?
   - Example 4: 5d SCFTs and toric geometry

Example 1: closure of minimal nilpotent orbits

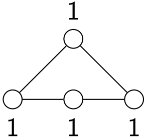
Notation for quivers:



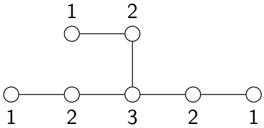
$\cong$



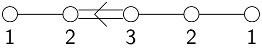
Fact: Affine Dynkin diagrams are magnetic quivers for closures of minimal nilpotent orbits:

$\mathcal{C}$ () $= \overline{\mathcal{O}}_{\min}(\mathfrak{sl}_4) =: a_3$

Example 1: closure of minimal nilpotent orbits



$$\mathcal{C}\left(\cdots \right) = \overline{\mathcal{O}}_{\min}(E_6) =: e_6$$



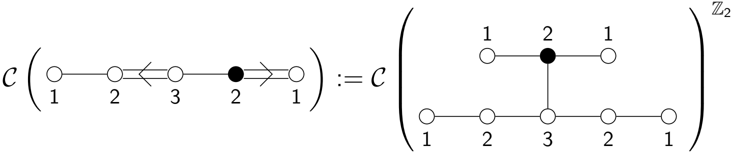
$$\mathcal{C}\left(\cdots \right) = \overline{\mathcal{O}}_{\min}(F_4) =: f_4$$

[Brylinski, Kostant 94]

Example 1: closure of minimal nilpotent orbits

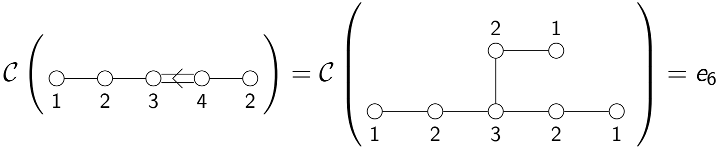
Remarks:

- $\mathcal{C}$ of non-simply laced quivers correspond to fixed points:



[Nakajima, Weekes 19]

- Non unicity of magnetic quivers



Outline

1. What are Magnetic Quivers?
   - Example 1: closure of minimal nilpotent orbits
   - Example 2: 4d $\mathcal{N} = 2$ SCFTs
2. What do they tell us?
   - Coordinate ring
   - The Stratification
   - Example 3: Slices in the Affine Grassmannian
3. How to derive them?
   - Example 4: 5d SCFTs and toric geometry

Example 2: 4d $\mathcal{N} = 2$ SCFTs

How to unambiguously specify a 4d $\mathcal{N} = 2$ SCFT?

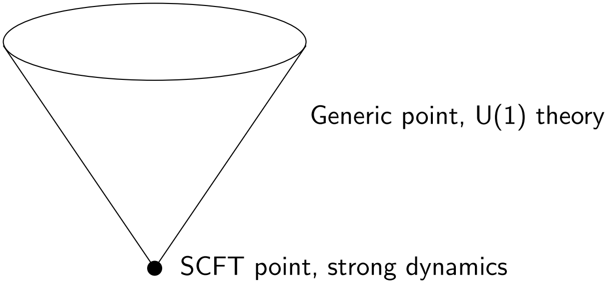
- Flavor symmetry
- Central charges
- Coulomb branch geometry
- Higgs branch geometry
- Seiberg-Witten curve / Integrable system
- Spectrum of BPS states
- Superconformal index
- 2d VOA

These data can be **derived** from realizations of the theory, or can be **constrained** from bottom-up.

Example 2: 4d $\mathcal{N} = 2$ SCFTs

Coulomb branch exploration:

- Space of theories graded by the *rank* : $r = \dim_{\mathbb{C}} \mathcal{C}$.
- Low-energy physics governed by the *singularity structure*. For $r = 1$:



- Distinct theories can share the same Coulomb branch geometry: the geometry has to be supplemented with the possible *mass deformations*.

Example 2: 4d $\mathcal{N} = 2$ SCFTs

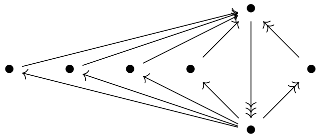
Classification of 4d rank 1 $\mathcal{N} = 2$ SCFT Coulomb branch geometries:

Flavor	CB geometry and deformation	$\Delta(u)$
E_8	$II^* \to \{I_1^{10}\}$	6
E_7	$III^* \to \{I_1^9\}$	4
E_6	$IV^* \to \{I_1^8\}$	3
D_4	$I_0^* \to \{I_1^6\}$	2
A_2	$IV \to \{I_1^4\}$	3/2
A_1	$III \to \{I_1^3\}$	4/3
$\emptyset$	$II \to \{I_1^3\}$	6/5
C_5	$II^* \to \{I_1^6, I_4\}$	6
$C_3 A_1$	$III^* \to \{I_1^5, I_4\}$	4
$C_2 U_1$	$IV^* \to \{I_1^4, I_4\}$	3
C_1	$I_0^* \to \{I_1^2, I_4\}$	2
$A_3 \rtimes \mathbb{Z}_2$	$II^* \to \{I_1^3, I_1^*\}$	6
$A_1 U_1 \rtimes \mathbb{Z}_2$	$III^* \to \{I_1^2, I_1^*\}$	4
U_1	$IV^* \to \{I_1^1, I_1^*\}$	3
$A_2 \rtimes \mathbb{Z}_2$	$II^* \to \{I_1^2, IV^*_{Q=1}\}$	6
$U_1 \rtimes \mathbb{Z}_2$	$III^* \to \{I_1, IV^*_{Q=1}\}$	4
$\emptyset$	$IV^*_{Q=1}$	3
C_1	$I_0^* \to \{I_2^3\}$	2

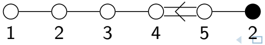
[Argyres, Lotito, Lü, Martone 18]

Example 2: 4d $\mathcal{N} = 2$ SCFTs

- Flavor symmetry : $\mathfrak{sp}(5)$, level $k = 7$.
- Central charges : $a = \frac{41}{12}$, $c = \frac{49}{12}$. Effective number of vectors $n_v = 11$, hypers $n_h = 27$.
- Coulomb branch geometry:
  - Complex Dimension = rank $r = 1$.
  - Scaling dimension $\Delta = 6$. Characteristic dimension $\kappa = 6$.
  - Singularity and deformation $II^* \to \{I_1^6, I_4\}$.
- Spectrum of BPS states [Cecotti, Del Zotto 14], [Del Zotto, García Etxebarria 22]



- Superconformal index : deduced from class S construction [Chacaltana, Distler 11]
- Higgs branch geometry:
  - Quaternionic dimension $d_{HB} = n_h - n_v = 24(c - a) = 16$
  - **Magnetic Quiver** :

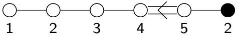
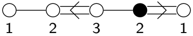
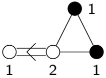
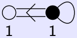
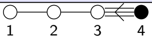
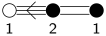
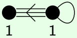
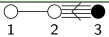
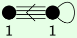
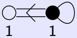


Example 2: 4d $\mathcal{N} = 2$ SCFTs

Flavor	$\dim_{\mathbb{H}}(HB)$	Magnetic Quiver
E_8	29	
E_7	17	Affine
E_6	11	
D_4	5	Dynkin
A_2	2	
A_1	1	Diagrams
$\emptyset$	0	

[Beem, Meneghelli, Rastelli 19]

[AB, Grimminger, Hanany, Sperling, Zafrir, Zhong 20]

Flavor	$\dim_{\mathbb{H}}(HB)$	Magnetic Quiver
C_5	16	
$C_3 A_1$	8	
$C_2 U_1$	4	
C_1	1	
$A_3 \rtimes \mathbb{Z}_2$	9	
$A_1 U_1 \rtimes \mathbb{Z}_2$	3	
U_1	1	
$A_2 \rtimes \mathbb{Z}_2$	5	
$U_1 \rtimes \mathbb{Z}_2$	1	
$\emptyset$	0	
C_1	1	

Plan

1. What are Magnetic Quivers?
   - Example 1: closure of minimal nilpotent orbits
   - Example 2: 4d $\mathcal{N} = 2$ SCFTs
2. What do they tell us?
   - Coordinate ring
   - The Stratification
   - Example 3: Slices in the Affine Grassmannian
3. How to derive them?
   - Example 4: 5d SCFTs and toric geometry

Magnetic Quivers

$$
\begin{tikzcd}
\boxed{\text{Generalized Quiver}} \arrow[r] \arrow[dr, dotted, "\text{Monopole Formula}"'] & \boxed{\text{Symplectic Singularity}} \arrow[d] \\
& \boxed{\text{Hilbert Series}}
\end{tikzcd}
$$

$$
\begin{tikzcd}
\boxed{\text{Generalized Quiver}} \arrow[r] \arrow[dr, dotted, "\text{Quiver Subtraction}"'] & \boxed{\text{Symplectic Singularity}} \arrow[d] \\
& \boxed{\text{Stratification}}
\end{tikzcd}
$$

[Cremonesi, Hanany, Zaffaroni 14]
[AB, Cabrera, Grimminger, Hanany, Sperling, Zajac, Zhong 20]

Outline

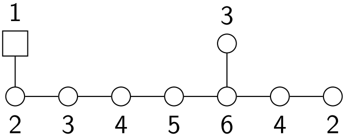
1. What are Magnetic Quivers?
   - Example 1: closure of minimal nilpotent orbits
   - Example 2: 4d $\mathcal{N} = 2$ SCFTs
2. What do they tell us?
   - Coordinate ring
   - The Stratification
   - Example 3: Slices in the Affine Grassmannian
3. How to derive them?
   - Example 4: 5d SCFTs and toric geometry

Coordinate ring

Simply laced quiver Q with

- Set of vertices V
- Set of (unoriented) edges $E \subset S^2(V)$
- Gauge group $U(n_v)$ for $v \in V$, total gauge group $G = \prod_{v \in V} U(n_v)$ of rank $r = \sum_{v \in V} n_v$ with Weyl group $W = \prod_{v \in V} S_{n_v}$.
- Flavor vertices $F \neq \emptyset$ with global symmetries $SU(n_f)$ for $f \in F$ and set of edges $E' \subset V \times F$

Example



Coordinate ring

A magnetic charge is an element $m \in \mathbb{Z}^r$. For H a subgroup of S_r and m a magnetic charge, we define the stabiliser

$$H(m) = \{g \in H | g \cdot m = m\} .$$

The conformal dimension $\Delta(m)$ is defined by

$$\begin{aligned} 2\Delta(m) &= \sum_{(v,v')\in E} \sum_{i=1}^{n_v} \sum_{i'=1}^{n_{v'}} |m_{v,i} - m_{v',i'}| + \sum_{(v,f)\in E'} \sum_{i=1}^{n_v} n_f |m_{v,i}| \\ &\quad - \sum_{v\in V} \sum_{i=1}^{n_v} \sum_{j=1}^{n_v} |m_{v,i} - m_{v,j}| . \end{aligned}$$

Monopole Formula gives graded dimension of coordinate ring $\mathbb{C}[X] = \bigoplus_{n\in\mathbb{N}} \mathbb{C}[X]_n$, with $\mathbb{C}[X]_2$ Lie algebra of isometries:

$$\boxed{\sum_{n\in\mathbb{N}} t^n \dim \mathbb{C}[X]_n = \frac{1}{|W|} \sum_{m\in\mathbb{Z}^r} \sum_{\gamma\in W(m)} \frac{t^{2\Delta(m)}}{\det(1 - t^2\gamma)}}$$

Coordinate ring

Hilbert series for the Higgs branch of the $\mathfrak{sp}(5)_7$ theory:

$$\frac{\begin{pmatrix} 1+2t+40t^2+194t^3+1007t^4+4704t^5+18683t^6+67030t^7+220700t^8+657352t^9+1796735t^{10} \\ +4540442t^{11}+10610604t^{12}+23011366t^{13}+46535540t^{14}+87887734t^{15}+155277056t^{16} \\ +257288236t^{17}+400453203t^{18}+585971786t^{19}+807195575t^{20}+1047954388t^{21} \\ +1282842123t^{22}+1481462886t^{23}+1615002952t^{24}+1662191888t^{25}+\cdots \text{palindrome} \cdots + t^{50} \end{pmatrix}}{(-1+t)^{32}(1+t)^{18}(1+t+t^2)^{16}}$$

Refined plethystic logarithm :

$$\begin{aligned} t^2 &: [20000] \\ t^3 &: [00001] \\ t^4 &: -[01000] \\ t^5 &: -[10010] \\ t^6 &: -[00200]-[20000]+[01000] \\ &\text{etc} \end{aligned}$$

Highest weight generating function [Hanany, Kalveks 16]

$$\mathrm{PE}\left[\sum_{i=1}^{4}\mu_i^2 t^{2i}+t^4+\mu_5(t^3+t^5)\right] \longrightarrow \text{Global form } \mathrm{Sp}(5)$$

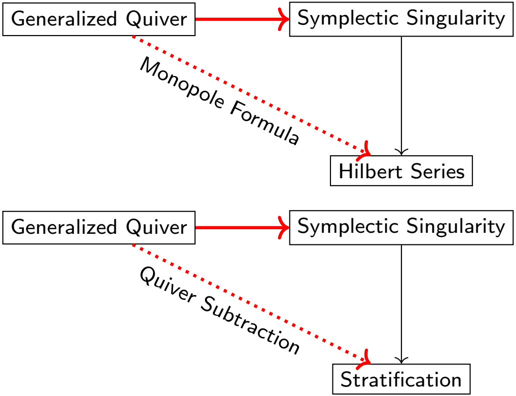
Outline

1 What are Magnetic Quivers?
- Example 1: closure of minimal nilpotent orbits
- Example 2: 4d $\mathcal{N} = 2$ SCFTs

2 What do they tell us?
- Coordinate ring
- The Stratification
- Example 3: Slices in the Affine Grassmannian

3 How to derive them?
- Example 4: 5d SCFTs and toric geometry

Magnetic Quivers



[Cremonesi, Hanany, Zaffaroni 14]
[AB, Cabrera, Grimminger, Hanany, Sperling, Zajac, Zhong 20]

The stratification

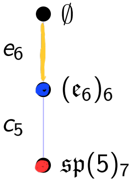
A CSS X admits a finite canonical stratification

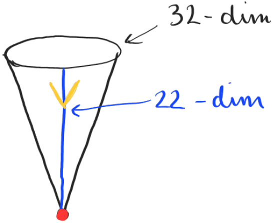
$$X = X_0 \supset X_1 \supset \cdots \supset X_n$$

where X_{i+1} is the singular part of X_i and the normalization of every irreducible component of X_i is again a CSS.

[Kaledin 04]

Stratification $\rightarrow$ Partial Order $\rightarrow$ Hasse diagram:





Physically, for Higgs branches it corresponds to (partial) *Higgsing*.

The stratification

QUIVER SUBTRACTION ALGORITHM:

INPUT:

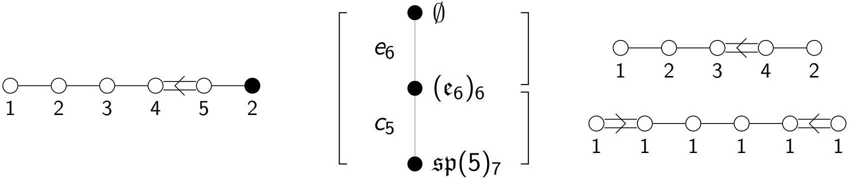
- A (generalized) quiver
- A list of **elementary** symplectic singularities with a corresponding magnetic quiver.

OUTPUT: The Hasse diagram of nested singularities.

[Cabrera, Hanany 18]

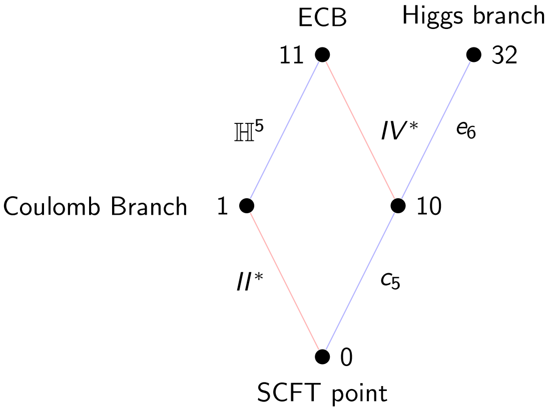
[AB, Grimminger, Hanany, Sperling, Zhong 21]

Example:



The stratification

Full moduli space of supersymmetric vacua also stratified.
Example: $\mathfrak{sp}(5)_7$ theory:

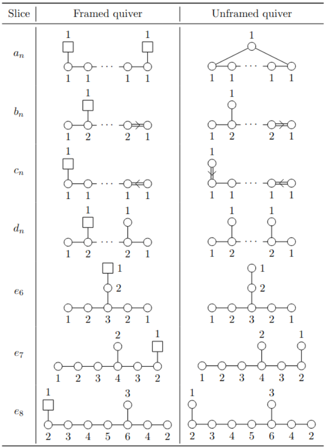


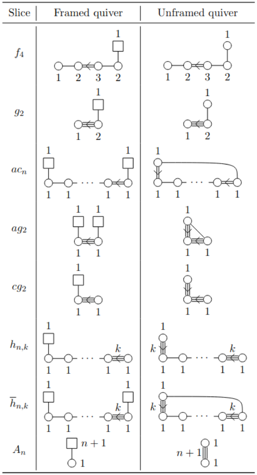
[Argyres, AB, Martone 19], [Argyres, Martone 21]

The stratification

Reductionist approach to Higgs branch geometries :

- What is the list of atoms (elementary symplectic singularities)?
- What are the rules to combine them?





Outline

1. What are Magnetic Quivers?
   - Example 1: closure of minimal nilpotent orbits
   - Example 2: 4d $\mathcal{N} = 2$ SCFTs
2. What do they tell us?
   - Coordinate ring
   - The Stratification
   - Example 3: Slices in the Affine Grassmannian
3. How to derive them?
   - Example 4: 5d SCFTs and toric geometry

Example 3: Slices in the Affine Grassmannian

The *Affine Grassmannian* Gr_G is an (infinite dimensional) ind-scheme whose underlying set can be represented as $G((t))/G[[t]]$.

Orbits under left action of $G[[t]]$ are labelled by dominant coweights of G.

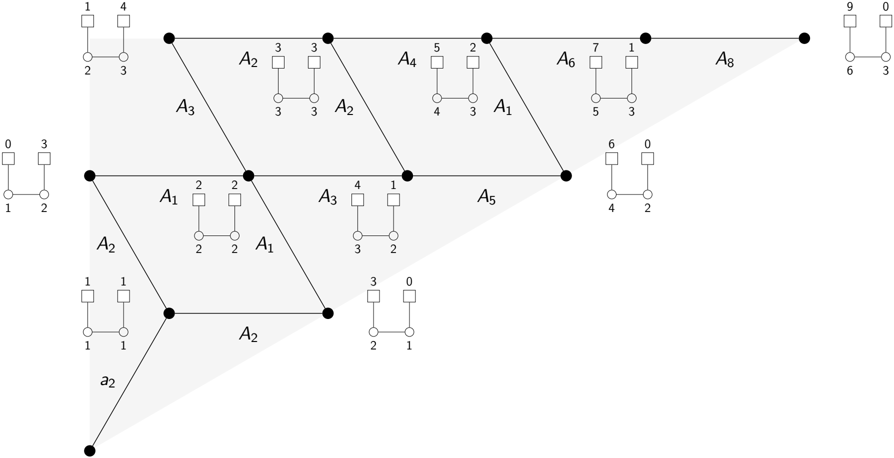
For $\lambda \leq \mu$ two dominant coweights of G, define a **finite dimensional** *transverse slice*, of dimension $\langle \mu - \lambda, \rho \rangle$.

If λ and μ are *adjacent*, the slice is an *elementary CSS*.

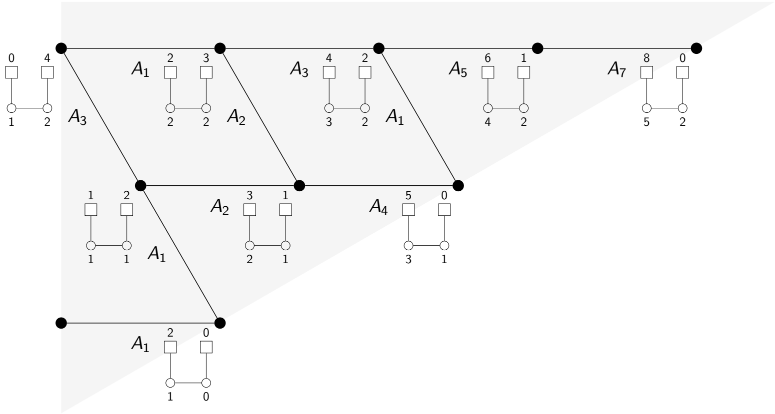
This is a source of elementary CSS.

[Bullimore, Dimofte, Gaiotto 15]
[Braverman, Finkelberg, Nakajima 16]

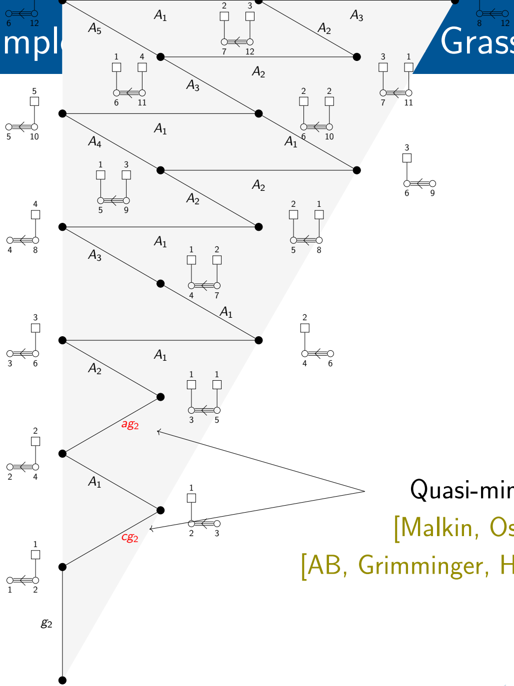
Example 3: Slices in the Affine Grassmannian



Example 3: Slices in the Affine Grassmannian



Exampl Grassmannian



Quasi-minimal singularities

[Malkin, Ostrik, Vybornov 03]

[AB, Grimminger, Hanany, Sperling, Zhong 21]

Plan

1. What are Magnetic Quivers?
   - Example 1: closure of minimal nilpotent orbits
   - Example 2: 4d $\mathcal{N} = 2$ SCFTs

2. What do they tell us?
   - Coordinate ring
   - The Stratification
   - Example 3: Slices in the Affine Grassmannian

3. How to derive them?
   - Example 4: 5d SCFTs and toric geometry

How to derive magnetic quivers?

Various methods can be used (in cooperation):

- Derivation from intersection numbers in brane systems [Cabrera, Hanany, Yagi 18], [AB, Cabrera, Grimminger, Hanany, Zhong 19], [Akhond, Carta, Dwivedi, Hayashi, Kim 20], [van Beest, AB, Eckhard, Schäfer-Nameki 20], [Akhond, Carta 21], [Sperling, Zhong 21] ...
- Deduction from known magnetic quivers (e.g. compactifications / twisted compactifications from higher dimension [Zafrir 16], [Martone, Zafrir 21])
- Computation of 3d mirror symmetry (e.g. for Argyres-Douglas theories [Giacomelli, Mekareeya, Sacchi 21], [Carta, Giacomelli, Mekareeya, Mininno 21], [Xie 21], [Dey 21], ...)
- Derivation from geometry of string backgrounds [Collinucci, Valandro 20], [Closset, Schäfer-Nameki, Wang 21], ...
- Guess based on knowledge of the chiral ring [Cabrera, Hanany, Zajac 18], [Arias-Tamargo, AB, Pini 21]
- etc...

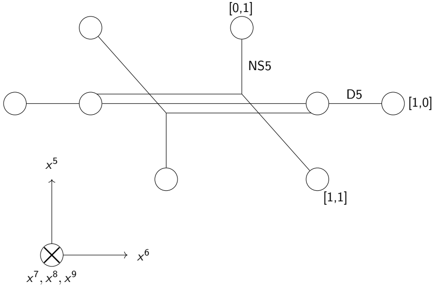
Outline

1. What are Magnetic Quivers?
   - Example 1: closure of minimal nilpotent orbits
   - Example 2: 4d $\mathcal{N} = 2$ SCFTs
2. What do they tell us?
   - Coordinate ring
   - The Stratification
   - Example 3: Slices in the Affine Grassmannian
3. How to derive them?
   - Example 4: 5d SCFTs and toric geometry

Example 4: 5d SCFTs and toric geometry

5d $\mathcal{N} = 1$ SCFTs are "non-Lagrangian"

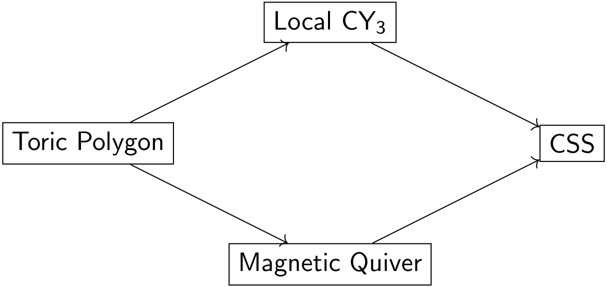
- Geometric engineering : M-theory on canonical threefold singularity.
- Brane systems : example of type IIB brane web



- Mixture of both : IIA with D6 on fibered ALE space.

[Intriligator, Morrison, Seiberg, Witten, Aharony, Hanany, Kol, Bergman, Rodrígez-Gómez, Zafrir, Del Zotto, Heckman, Jefferson, Katz, Kim, Vafa, Xie, Yau, Closset, Schäfer-Nameki, Wang, Hayashi, Yagi, ...]

Example 4: toric geometry



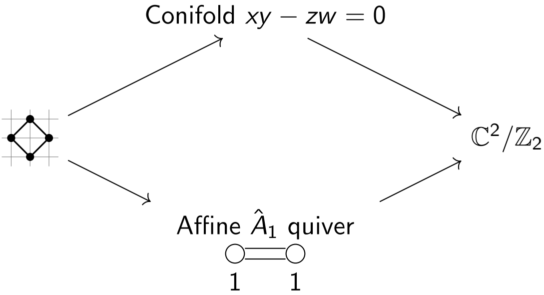
- The CSS associated to a local CY_3 Y is the Higgs branch of the 5d $\mathcal{N} = 1$ SCFT obtained from M-theory on $\mathbb{R}^{1,4} \times Y$.
- The MQ associated to a toric polygon is given by Minkowski decomposition / brane web tropical intersections.

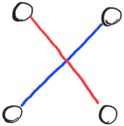
[Cabrera, Hanany, Yagi 18]

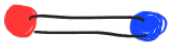
[van Beest, AB, Eckhard, Schäfer-Nameki 20] $\rightarrow$ Mathematica

Example 4: toric geometry

Simple example:

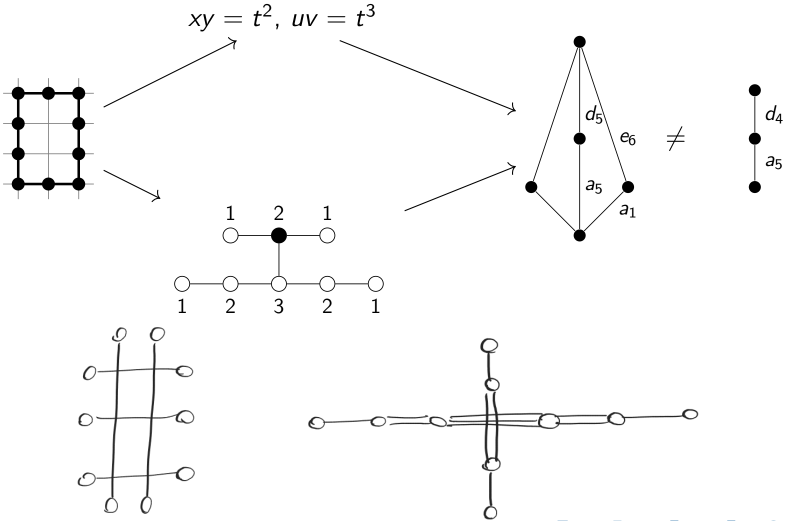






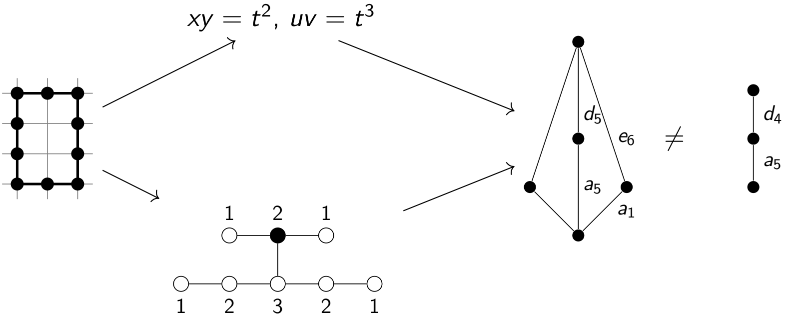
Example 4: toric geometry

5d $\mathcal{N} = 1$ SCFT corresponding to $\mathrm{SU}(3)_0 + 6\mathrm{F}$:



Example 4: toric geometry

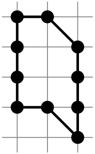
5d $\mathcal{N} = 1$ SCFT corresponding to $\mathrm{SU}(3)_0 + 6\mathrm{F}$:



To be compared with the gauge theory (contribution of massless instantons):

- Dimension jump ($24 \neq 20$)
- Symmetry enhancement ($A_5 \oplus A_1 \oplus A_1 \neq A_5$)

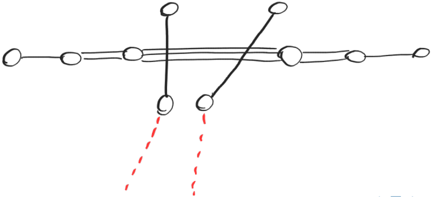
5d $\mathcal{N} = 1$ SCFT corresponding to $\mathrm{SU}(3)_1 + 6\mathrm{F}$:



???

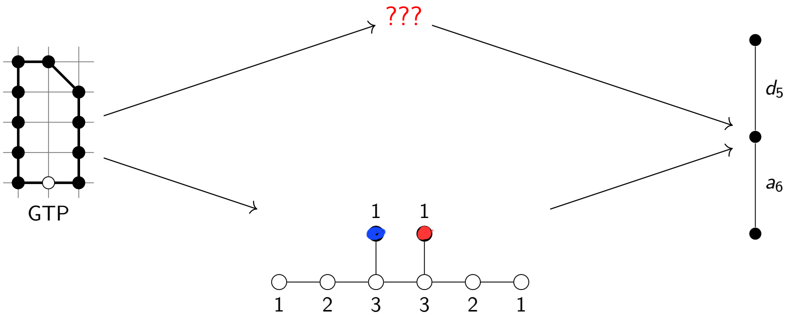
???

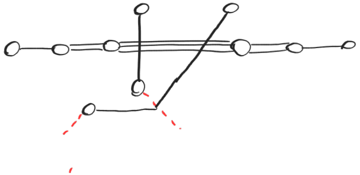
???



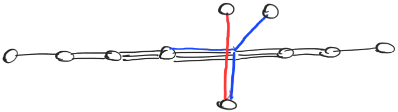
Example 4: toric geometry

5d $\mathcal{N} = 1$ SCFT corresponding to $SU(3)_1 + 6F$:



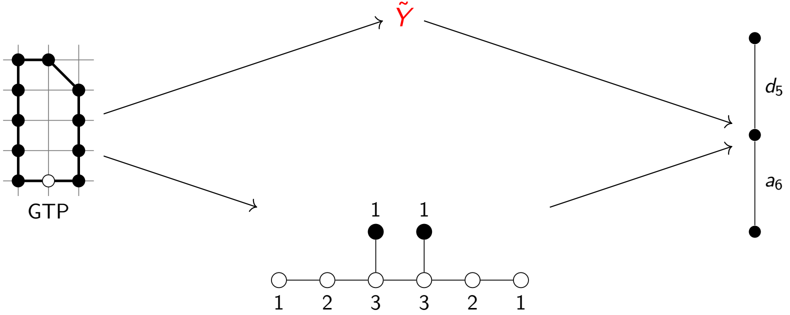


[Benini, Benvenuti, Tachikawa 09]



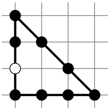
Example 4: toric geometry

5d $\mathcal{N} = 1$ SCFT corresponding to $SU(3)_1 + 6F$:



[Benini, Benvenuti, Tachikawa 09]

Need extension of toric geometry.
$\tilde{Y}$: non-toric deformation of toric Y.
Example: $xyz = w^n + \alpha w^{n-2} x$ for $\mathbb{C}^3/(\mathbb{Z}_n)^2$.



[AB, Collinucci, Schäfer-Nameki, Süß, WIP]

Outlook

Magnetic Quivers are a way of defining Conical Symplectic Singularities.

For certain physically / mathematically interesting CSS, the MQ is the only (?) available description.
E.g. hyperKähler Implosion Spaces [AB, Dancer, Grimminger, Hanany, Kirwan, Zhong 21]

The knowledge of the MQ gives access *algorithmically* to CSS data:

- Dimension
- Global symmetry
- Structure of the coordinate ring
- Singular stratification

For Higgs branches of QFTs this characterizes their **phase structure** and how theories are connected via **generalized Higgsing** and **RG flow**.

Outlook

Future directions and open problems:

- What is the scope of magnetic quivers? Various extensions of the notions have already been proposed. What is the generic magnetic "object"?
- What other information can be extracted from magnetic quivers? E.g. HyperKähler metric?
- Is there a possible "bootstrap" approach?
  - Classification of possible elementary slices?
  - NEW! CSS found by [Bellamy, Bonnafé, Fu, Juteau, Levy, Sommers 22] $\rightarrow$ Appears in 6d $\mathcal{N} = (1,0)$ SCFTs, MQ in [AB, Grimminger, Juteau, Levy, WIP]
  - How do slices combine in general?

Thank you for your attention!